ROMAN SUSZKO AND SITUATIONAL IDENTITY

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1. Introduction

At the start of his first paper dealing with the quantified sentential calculus with identity, Roman Suszko writes (Suszko [1968: 8-9]):

> Consider …
>
> (1)   Some situations are not facts
>
> It has the same grammatical structure as the following sentence:
>
> (2)   Some philosophers are not logicians
>
> Both sentences (1) and (2) are existential sentences. But there is a very deep difference between them. The terms «philosopher» and «logician» in (2) are unary predicates. The terms «situation» and «fact» are not predicates. They are unary sentential connectives like the word «not» which converts any sentence ϕ into a negation: not-ϕ. To see this, let us make the first step in formalizing (1) and (2). We write:
>
> (3)   for some *p*, *Sp* and not-*Fp*
>
> (4)   for some *x*, *Px* and not-*Lx*.
>
> The letter *p* is a sentential variable and the letter *x* is a nominal variable. They are bound above in (3) and (4) by the existential quantifier: «for some». The symbols *P* and *L* are unary predicate and the letters *S* and *F* are unary sentential connectives like the connective «not». The difference between sentences (sentential variables, sentential formulae) and names is very deep and fundamental in every language. It must be observed in any rigorous thinking. However, natural language leads sometimes to confusion on this point. Having in mind the categorical difference mentioned above we consider the sentence (1) quite as legitimate as the sentence (2). Moreover, both sentences (1) and (2) are true because:
>
> 1.   It is a situation that London is a small city but it is not a fact.
>
> 2.   Dr. B. W. is a good philosopher but he is not a logician.

The quantifier 'for some' is used in accord with certain inference rules. It is regrettable that we have fallen into the habit of calling that quantifier the *existential* quantifier, for this makes it look as if logic settled something ontological. Some authors, sensitive to this circumstance, have spoken instead of the *particular* quantifier, characterized in terms of its role in inference. That seems to me to be a sound way of speaking, and it is one I shall throughout adopt.

Suszko is inclined to contrast two particular quantifications

(∃x)(x is a philosopher & x is not a logician),

(∃p)(it is a situation that p & it is not a fact that p)

by pointing to the difference between a sentence and a proper name.

But isn't this only a *grammatical* difference?

Quine would say these sentences *both* assert the existence of some object or entity, arguing in something like the following way: Since the result of putting 'Dr. B. Wolniewicz' in for 'x' in 'x is a philosopher & x is not a logician' is an *instance of* and *logically entails* the first quantification, 'Dr. B. Wolniewicz' there occurs *referentially*, its *referent* being some *object*. Similarly, since the result of putting 'London is a small city' in for 'p' in 'It is a situation that p and it is not a fact that p' is an instance of and logically entails the second quantification, 'London is a small city' there occurs referentially, its referent being some object. So, though 'Dr. B. Wolniewicz' is grammatically a proper name and 'London is a small city' is grammatically a sentence, each is *semantically* a term of reference with some object as its referent. Quine [1961: 118].

The guiding principle of this argument is that terms accessible to positions accessible to a variable of quantification are all *referential* and their referents, if any, are *objects*.

What *supports* this principle?

Consider the following answer: What shows that each of these terms (despite their having grammatically different types of occurrence) *refers* to some *object* is this: the kind of *semantical account* we must provide for the quantifications which bind variables in positions available to these terms.

For example, one type of semantical account would assign e.g., human beings as referents for names and nominal variables and situations or propositions or truth-values as referents for sentences and sentential variables. Thus, Ryszard Wójcicki provides such an account for Suszko's quantified sentential calculus with identity. Wójcicki [1984: 326, 333-335]. And M. J. Cresswell provides such an account for propositional identity. Cresswell [1967: 284].

But *how* is the possibility of this sort of semantical account of quantification supposed to show that in its occurrence in 'If it is a situation that London is a small city, then (∃p) it is a situation that p' 'London is a small city' *refers to* something — be it a proposition or a situation or a truth-value or anything else?

One answer would be that it is possible to give an adequate semantical account of quantification *only* in referential terms, and that there *must* be an adequate semantical account of quantification.

Particular quantification is a certain sort of operation within language. The question is when *that* operation forms sentences in assenting to which we commit ourselves to the existence of things.

One answer that suggests itself is to go beyond the inference role of the particular quantifier and find out its semantical role. Its semantical role should be made clear in a truth theory of the language under discussion.

So, I want to ask whether the general sort of semantical account of quantification I have sketched is the *only* one possible. Why not a *non*-referential semantical account of quantification of those quantifications which bind variables in for sentences? I here propose such an account for the quantified sentential calculus propounded by Suszko. I think that such an account is more in keeping with Suszko's remarks quoted at the beginning of this paper

than is Wójcicki's semantical account. Suszko was attempting to set out the ontology of the *Tractatus*. For such an end it would not do to treat grammatically proper names and sentences the same way semantically. For Wittgenstein, while situations are made up of objects, they are not objects themselves. I do not think Suszko's commentators have appreciated the point that for Suszko grammatically proper names and sentences ought not to be treated the same way semantically. For instance, Grezegorz Malinowski writes, «The identity connective … was introduced by Suszko to express coincidence and other referential relations between sentences.» Malinowski [1985: 21].

Quine observes that there are non-referential accounts available for such varieties of quantification. For example, quantifications binding variables in for sentences and clauses whose only modes of combination are truth-functional can be introduced, and thus also eliminated, in terms involving no binding of variables in positions apt for sentences. Hence, an account of such quantification need involve only such elements of reference as may be required by an account of the underlying truth-functionally composed sentences and clauses. Quine [1961: 118]. Also see Church [1956: 151-154]. Further, in some cases we may so quantify as to find it *necessary* for the truth of a particular quantification that it have some true instance. Such quantification is called *substitutional* and its account need involve only such elements of reference as may be required by an account of the sentences which serve as instances of quantifications.

I do not here controvert these claims. But I do note that the first claim is inapplicable to the sort of system Suszko investigates. For that system includes '=' as a sentential connective. And that connective is not truth-functional. As to the second claim I note that those of our particular quantifications which bind variables in positions appropriate for sentences include ones for the truth of which we do *not* require that they have some true instance. For example, 'Some situations are inexpressible' or '($\exists$p) (it is a situation that p & that p is inexpressible)'.

So when I consider quantifications of the type which bind variables in for sentences, I will take it that such quantifications are not to be construed substitutionally.

2. Some Theses of the Tractatus

Suszko was attempting to set out the ontology of the *Tractatus*. The language Suszko used included *(a)*the ordinary truth-functional calculus, enriched with quantifiers binding variables standing for sentences, and with an identity-function with sentences as arguments; *(b)* the ordinary theory of quantification applied to the special quantifiers; and *(c)* ordinary laws of identity applied to the special function. I propose to give a semantical account of the special quantifiers and the special function. I intend it to be non-referential. I do not intend to follow Suszko in setting out the ontology of the *Tractatus*. But I do intend it to incorporate some of that work's theses.

Here are some theses from Wittgenstein's *Tractatus Logico-Philosophicus*:

1. There are elementary sentences.

2. Elementary sentences are combinations of words.

3. If a word can occur in an elementary sentence, then it names some object.

4. The meaning of a word that can occur in an elementary sentence is the object it names.

5. An object named by a word that can occur in an elementary sentence is *simple*.

6. No elementary sentence names anything.

7. Each elementary sentence *says* that something is the case.

8. It is by means of the fact that its names stand to one another in certain relations within the elementary sentence that an elementary sentences *says* that something is the case.

9. An elementary sentence is true if what it *says* is the case, *is* the case, and is false if what it *says* is the case, *is not* the case.

10. What an elementary sentence *says* is the case *can* be the case, and also *can* fail to be the case.

11. If a sentence says something that *can* be the case, and which *can* also fail to be the case, then it is a truth-function of elementary sentences.

12. A non-elementary sentence that *fully* articulates what it says consists of names of the kind which can occur in elementary sentences combined into a whole with terms of logic.

13. No logical term names anything.

14. If a sentence does not say which *can* be the case, and which *can* fail to be the case, then it says nothing at all.

15. What a sentence says is something which can be expressed in various ways corresponding to various attitudes and interests, for example, as when we *ask whether* or *order it to be the case that* or *assert that* the door is closed.

I do not think any semantical account, even one which incorporates *Tractatus* elements, should try to incorporate all of these ideas. Thesis 11 is definitely false. Thesis 14 leads to the idea that neither necessary truths nor necessary falsehoods say anything. Since a true (false) sentence is one which says something true (false), this idea is incoherent. (This is argued by Hugly and Sayward in [1999]).

What of the other theses? First, there has been much thoughtful criticism of idea that the meaning of a name is the object it names. Second, since predicates must occur in elementary sentences, predicates must be names. How plausible is this? Finally, can anyone come up with a single example of an elementary sentence? Or a *simple* object? These are dubious ideas which Wittgenstein himself effectively criticized in the first sixty sections of the *Investigations*.

If we drop these dubious ideas we are still left with a lot. Every atomic sentence of a truth-functional language says something. What is said at this level determines what gets said by every sentence which is a truth-functional compound of the atomic sentences. A sentence is true if what it *says* is the case, *is* the case, and is false if what it *says* is the case, *is not* the case. If we now add a non-extensional connective such as 'John believes that' to the language, the result of appending that connective to a truth-functional sentence is true just in case John believes what is said by that sentence. These are ideas suggested by the *Tractatus*.

One of Suszko's thoughts is to add '=' to the language as a sentential connective. Then, for any two truth-functional sentences ϕ and ψ, $(\phi = \psi)$ is true just in case what is said by ϕ is the same as what is said by ψ. A second of Suszko's thoughts is to add sentential quantifiers to the language. Since sentences are not names (a thought shared by Wittgenstein and Suszko and many others), the sentential quantification introduced is not to be treated as

referential quantification. Finally, Suszo reads the sentential variables as taking situations as values.

Now the task is to put all of this to work to get a semantics for the resulting language.

3.Semantics

In what follows upper case letters 'S' and 'T' will function as sentential variables in the metalanguage, ranging over situations. The lower case letter 'α' will function as a variable in the metalanguage which ranges over sentential variables of the object language.

The language of truth-functional logic will be extended by the addition of the particular quantifier '∃' which binds the sentential variables and by the addition of two sentential connectives, '=' and 'B', which are not truth-functional. The first of these is a binary connective read as 'That __ is the same situation as that …'. The second is a unary connective read as 'John believes that __'.

Truth-functional formulas are understood in the usual way, using brackets, sentential variables, and a binary connective for the stroke function.

The formulas of the extended language are as follows (bold faced type functions as quasi-quotation):

(a)   Any truth-functional formula is a formula.

(b)   For any truth-functional formulas ϕ and ψ, **(ϕ = ψ)** is a formula.

(c)   For any truth-functional formula ϕ, **Bϕ** is a formula.

(d)   For any formulas ϕ and ψ, **(ϕ | ψ)** is a formula.

(e)   For any variable α and formula ϕ, **∃α ϕ** is a formula.

(f)   Nothing else is a formula.

A model for the language is an assignment of a situation to each variable of the language. More precisely, for each sentential variable α there is one situation S such that a model M of the language interprets α to say that S.

Each truth-functional formula says something relative to M (M-says something): (i) for any situation S, a sentential variable M-says that S if and only if M interprets it to say that S; (ii) for any situations S and T, and for any truth-functional formulas ϕ and ψ, **(ϕ | ψ)** M-says that neither S nor T if and only if ϕ M-says that S and ψ M-says that T.

Truth in M then runs as follows:

(g)   A truth-functional formula ϕ is true in M if and only if, for some situation
         S, ϕ M-says that S and it is the case that S.

(h)   For any truth-functional formulas ϕ and ψ, **(ϕ = ψ)** is true in M if and only
         if, for some situation S and for some situation T, ϕ M-says that S and ψ M-
         says that T and the situation that S is the same as the situation that T.

(i)   For any truth-functional formula ϕ, **Bϕ** is true in M if and only if, for some
         situation S, ϕ M-says that S and John believes that S.

(j)   For any formulas ϕ and ψ, (ϕ | ψ) is true in M if and only if neither ϕ nor
           ψ is true in M.

(k)   For any variable α and formula ϕ, ∃α ϕ is true in M if and only if ϕ is true
           in some model M* which differs at most from M in the interpretation of α.

The stipulation that, for any sentential variable, a model assigns *one* situation to that variable, assures that no truth-functional formula is ambiguous. That is, it follows that, for any truth-functional formula ϕ, there is one situation S such that ϕ says that S. This is a necessary requirement. For suppose that, for some truth-functional formula ϕ, ϕ said something that is the case and something else that is not the case. Would ϕ be true or false?

It may be objected that I have used connectives for situational identity and belief to give a semantical account of a language which includes such connectives. But that would be like objecting to the use of a connective for disjunction to give a semantical account of a language which includes a connective for disjunction.

It may be objected that I have used sentential quantification to give a semantical account of sentential quantification. But why is this objectionable? Referential quantification is used to give a semantical account of referential quantification in first order logic. Nobody objects. So what is objectionable in using sentential quantification to give a semantical account of sentential quantification?

4.Ontological Commitment

What it is for someone to be a realist about electrons, or numbers, or properties, or situations, and so on? I find it natural to answer this question thus: It is for that person to be committed to the existence of electrons, or numbers, or properties, or situations.

I agree that existence is expressed by such particular quantifiers as

There are…

Something…

There is something which…

There exists something which…

I disagree that whenever one uses such quantifiers to make assertions one asserts existence.

There is at least a *prima facie* case that not all uses of 'there is' and the like are ontologically committal. Consider the following sentences:

There is a question whether you are reluctant to speak up in this class.

There is more to life than philosophy.

There is a mistake in the derivation!

There is a way of putting the point that won't hurt his feelings.

That a person assents to such sentences as these does not *itself* show that he or she is ontologically committed to the existence of questions, that which is more to life than philosophy, mistakes in arguments, and ways of putting points. There are indefinitely many such examples.

The question is when the operation of particular quantification forms sentences in assenting to which we commit ourselves to the existence of things.

I suggest the following answer: a particular quantifier asserts existence just in case it is a referential quantification (a quantification whose bound variable takes terms of reference, for example, names, as substituends).

Given (k) of the truth definition and the fact that neither '=' nor 'B' are truth-functional, sentential quantification is neither substitutional nor eliminable. Is it referential, then? Do the sentence letters of the object language take objects as values thereby becoming terms of reference? Unless there is some good argument that sentential quantification must be referential if not substitutional or eliminable, I have no reason to suppose the sentential variables of this object language take objects as values or function as terms of reference.

At the end of the second section I said that Suszko reads the sentential variables as taking situations as values. I intended the semantical account to incorporate this feature. But I see nothing about this that has ontological relevance.

We are inclined to speak of the values of the variables, and to speak of what the variables range over. In this connection we distinguish the values of a variable from its substituends, the expressions which can replace it.

In one case the values of the variables are people and the variables range over people. In another case the values of the variables are situations and the variables range over situations. In both cases the variables are letters. Now what is it to have a value — to have a person or a situation as a value? And is having a value the same in both cases?

In terms of models and truth relative to models, the idea of *ranging over* is expressed by the role variant models play in defining truth relative to a model for the quantifications. A variable has one value relative to one model, another value relative to one of its variants, a third value relative to another of its variants, and so forth. This holds independently of what it is for a variable to have a value or a value of this or that sort. So *ranging over* can be said to be the same for sentential variables and variables for people.

A natural picture for *ranging over* would be that of a gesture of hand in respect to a group of people. And that carries with it a picture of having a value: we think of the people as values, and the gesture over them is like an ever so rapid pointing at each. But the sober fact is that there are many variant models and in each there is a particular variable with a particular value. That is all there is to *ranging over*.

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