KATHOLIEKE
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Institute of Philosophy

FUZZINESS AND THE SORITES PARADOX

From Degrees to Contradictions

Promoter: Professor Leon Horsten

Dissertation presented to fulfill the requirements for the degree of Doctor (Ph.D.) in Philosophy
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esta nueva lógica ... ese razonamiento heraclídeo en el cual las conclusiones no parecen congruentes con sus premisas...

no rechazo ya la paradoja como expresión de la verdad, pues fácilmente se me alcanza que... un gran descubridor de verdades será, en ocasiones, un gran forjador de paradojas.

Las verdades vitales son siempre paradójicas.

Nunca estoy más cerca de pensar una cosa que cuando he escrito la contraria.

¿Dijiste media verdad?
Dirán que mientes dos veces
si dices la otra mitad.

Antonio Machado

*****

this new logic ... this heraclitean reasoning in which the conclusions seem not to be consistent with their premises...

I do not reject paradox as the expression of truth, for I easily understand that... a great discoverer of truths will be, sometimes, a paradox inventor.

Vital truths are always paradoxical.

I am never closer to think something than when I have written the contrary.

Did you say half truth?
I will be said that you lie twice
if you say the other half.

Spanish quotes by Philip G. Johnston [2002] *The Power of Paradox in the Work of Spanish Poet Antonio Machado*, Lewinston (NY), The Edwin Mellen Press (pp. 184, ii, 9, 7, 77, resp.) The English translation in mine.

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PREFACE

Writing philosophy in a scholarly manner is not an easy task, at least when one attempts to be rigorous, following the standards of analytic philosophy, that is, in a responsible manner, adducing reasons for one's beliefs. I am not referring to just the style or form. What poses the difficulty is that the very subject matter sometimes is obscure and eludes our comprehension and our desire for clarity. Two of those tough philosophical problems are fuzziness and the sorites paradox. They have puzzled many great minds nowadays and in the past.

That both topics are not easy can perhaps be appreciated by considering the mood, or the changes of opinion of various professional philosophers of different schools. For example, the most famous agnosticists have personal histories to tell. Thus, Timothy Williamson opens the Preface of his *Vagueness* (p. xi) with this confession:

> This book originated in my attempts to refute its main thesis... For years I took this epistemic view of vagueness to be obviously false, as most philosophers do. ...Roy Sorensen's intriguing book *Blindspots*, which includes a defence of the epistemic view... did not persuade me; I could not see what makes us ignorant...

And Roy Sorensen (2001: 17) sincerely discloses his feeling of discomfort with respect to his own theory:

> Even though I accept epistemicism, I have signs of also disbelieving it. Despite the new respectability of epistemicism, I continue to be embarrassed by its characteristic tenet. I cannot suppress a nervous smile when asserting that the entry of one more individual into an auditorium might make it crowded. ... This enduring embarrassment is a disturbing sign that I fail to believe in sharp boundaries. Even worse, it has the air of a lie.

So, while Williamson was skeptic of epistemicism before he was able to see how to explain the ignorance involved, Sorensen continues to experience some uneasiness with it, even after seventeen years of professing the doctrine!

On other quarters, some nihilist authors have abandoned their nihilist convictions after some years. This is the case with Peter Unger. As we will see later in Chapter 5, in 1979, he defended that the sorites argument was a reductio ad absurdum of the supposition that there are ordinary objects, like a table, a stone, a person, etc. But in 1990, he thinks that the reductio is directed rather towards the «highly appealing» major premise, which is then considered false (192). Unger says that perhaps he was not completely clear about the ultimate point of his earlier papers, partly through his «own confusion» (332). «I had... both truly enormous logical and philosophical deficiencies» (*Ibid.*). Likewise, Mark Heller retracted his nihilism. In 1990, he straightforwardly argued for typical nihilist theses, such as that none of the physical objects exist (75); that the objects of the standard ontology do not exist (107-8); and that the world is not the way we think it is (69). However, in 1996 (185, n. 7), he avows that he [merely] leaned «towards the view that most of our everyday utterances are false even when appropriate. That now seems an extravagance».

Another example of change of view is Crispin Wright, who initially criticized Putnam's intuitionism, until he found how to answer to the objections (2003c: 96, n.). His evaluation of epistemicism, and the role played by the 'definitely' operator have also experienced some shifts (Cfr. his 1994, and 2001: § 7: 87-91).

A final example of disavowal of a position formerly held is Diana Raffman. She has defended contextualism in (1994b and 1996), but developed a non contextualist theory later (2005: 248).

I take these personal vacillations to hint to the complexity of the subject. «The topic of vagueness is a very difficult one» as Michael Dummett has said (quoted by Termini: 205, without an indication of the source). Michael Tye corroborates this: «whichever way we turn in our attempt to understand vagueness... we quickly become enmeshed in difficulties. Of all the philosophical mysteries, vagueness is surely one of the deepest» (1994: 19). The matter is so much intricate that Mark Sainsbury has told me in a personal communication that he has drowned in the quicksand.

Against this background, it would sound pretentious to claim that, after studying the topic for four or five years, I have come to have a clear view of it. Prudence counsels caution. I wish I could step into the terrain without offending the sensibilities of other thinkers. Unfortunately, I have chosen not to be neutral and, instead, I will advance the cause of certain non classical logics. No doubt, for many philosophers, my program will have all the appearances of being wrongly headed from the start. I am fully aware that the topic is hot and delicate. Yet, I have tried to be moderate, and balanced as far as possible. I submit my findings to the much better informed judgement of the reader. I should be prepared to defend the main contentions made, or be willing to admit that I have made a mistake. An open attitude seems to be just expected.

Finally, I want to express my sincere thanks to people who have helped me in one way or another. First and foremost, to my promoter, Prof. Leon Horsten, whose keen advice has prompted many improvements, throughout. To Lorenzo Peña, for suggesting me how to tackle with hard questions. His works have been a source of inspiration, and whose theory is here developed. To Kenton Machina, Nicholas Smith, Laurence Goldstein, Dominic Hyde, Matti Eklund and Guido Vanackere, who have sent me either critical comments or clarifications of their views in private correspondence. And to Francesco Paoli and Prof. Marnix Nuttin , for a lively interchange of ideas.

Thanks of another nature go to my wife Ana and daughters, Ana Gabriela and Antonella, who all have created an environment where daily live has been easier and happier. And, last but not least, to our special friend Danielle Van der Weeen, for making us feel at home being abroad.

Leuven, June 30, 2006

CHAPTER 1

INTRODUCTION

0.- Purposes and Importance of the Research

The topic of the present work falls primarily in the area of philosophy of language. Yet, it also can perhaps be classified as belonging to the field of philosophy of logic, since one of the problems it will examine is (1) what logic is the most adequate to represent and account for the phenomenon of fuzziness, as it appears in reality, our language and thought. Furthermore, this first problem does not come alone, but brings with it a second one, namely, (2) how best to cope with the philosophical aspects involved in the sorites paradox.

Concerning these two difficulties, my intentions are double. The first goal is negative, or destructive, in the sense that my aim is to criticize the standard, bivalent and truth-functional logic (CL, from now on) for being inadequate and deficient in solving both questions. But beside that, there lies the positive end of showing that the approach proposed to replace classical logic, a special blend of many-valued and paraconsistent logics, is most possibly in a better position than other alternatives to deal with the problems of fuzziness and the sorites. Indeed, the solution to at least one version of the paradox will consist in declaring invalid the form of the argument, to wit, disjunctive syllogism for the weak negation. Apparently, this is a radical move. However, it is important to make it clear from the start that the reform of CL here advocated consists of a demand for an extension[1] of the scope of its jurisdiction, rather than a reduction or curtailment of its power.

Thus, the two topics to be investigated are taken to be the main motivations to go beyond CL, towards degrees of truth and contradictions. But bear in mind that, since the whole of CL is incorporated into the new system, there is nothing to lament. Quite on the contrary, this step forward constitutes a necessary enrichment of the received logic.

Therefore, we are going to walk through a path leading to a change of logic. Of course, this has deep consequences. Given that logic sets the limits of what is rational, this notion itself must also be expanded. Indeed, the scope of transformation includes the realm of thinking, for logic puts boundaries to whatever is imaginable or thinkable. Depending on what sort of logic we espouse, the range of thought is going to be narrowed or widened. And there are ontological consequences also.

The dissertation, thus, aims at describing one rationale behind a particular trend of many-valued and paraconsistent logics.

1.- Preliminaries

Before beginning, some points require our previous attention. First, I present a sketch of the logic underlying the research, and an examination of different versions of the principles of bivalence, excluded middle and non contradiction. Later I expose some reasons why I prefer to use the word 'fuzziness' instead of the more common 'vagueness'. And, finally, I indicate that the general approach of this dissertation is semantic, keeping pragmatic considerations to a minimum.

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[1] Indeed, the systems here used, *Aj* and *Aq*, are *strict extensions* of CL, i.e., they keep every tautology and theorem as well as every rule of inference of CL provided that its negation sign be read as a strong negation. In other words, *Aj* and *Aq* are conservative extensions of CL. But of course, there are much more truths and rules to be added.

1a.- Overview of the Logical System to Be Used

The conventions concerning the logical notation used in this work are those of Alonzo Church. That is, roughly, a dot immediately after a functor means that its right member is everything to the right of the functor. When the right member is something shorter than the rest of the formula, parentheses are used. A connective is associative to the left, which means that its left member goes as far as the beginning of the formula, unless there is in its left side another functor with a reinforcing dot, in which case, the left member of the functor in question goes till the dot.

The logical system used here has been set up by Lorenzo Peña mainly in [1991] and [1993a]. Both the sentential calculus, *Aj*, and the quantificational one, *Aq*, are infinitely valued and paraconsistent. It is important to note that, contrary to what usually happens with other non classical logics, the systems of the family *A* are strict extensions of the classical logic, i.e., all the theorems and inference rules of CL are kept in the new system, provided that the classical negation, '¬', is read as 'not at all'. A brief semantical presentation of what is strictly needed is offered in the remaining of this section.

The novelty of the propositional calculus is its introduction of several new functors. Beside the classical (strong) negation, material conditional and biconditional, *Aj* contains at least two functors of affirmation, a weak negation, an implication and an equivalence functor. Let me characterize each.

First, *Aj* allows us to make nuanced affirmations. The functors 'Hp' and 'Lp' both assert that "p" is true, the difference being that 'H' assigns only complete truth, whereas 'L' assigns truth to some degree, partial or absolute. "Hp" is read as "it is totally true that p", while "Lp" means that "p is more or less true", "p is to some extent true", etc. They obey the following laws:

$$/Lp/ = \begin{cases} 1, \text{ if } /p/ > 0, \\ 0, \text{ otherwise.} \end{cases} \qquad /Hp/ = \begin{cases} 1, \text{ if } /p/ = 1, \\ 0, \text{ otherwise.} \end{cases}$$

See their truth table below. The two slashes flanking a formula "p" represent its truth value.

Second, the most important distinction I shall make is that between two sorts of negation: '¬', and '∼'. The former is the classical one, absolute, total or *strong negation*, over-negation or super-negation, the latter being the simple, plain, natural, or *weak negation*. We will read '¬' as 'not at all', 'it is completely false that', and the like, while '∼' will be read simply as 'not', 'it its false that', etc., without any intensifying qualification. The semantical definition of "¬p" is that it takes the value 1 whenever "p" gets the value 0, taking the value 0 otherwise, whereas the truth value of a sentence of the form "∼p" is equal to 1 minus the truth value of "p". The difference between both negations can be appreciated in the second and third columns of the following truth table, for a pentavalent logic.

p	∼p	¬p	Lp	Hp
+1	0	0	1	1
±¾	¼	0	1	0
±½	½	0	1	0
±¼	¾	0	1	0
-0	1	1	0	0

The signs '+', '-', '±', prefixed to the truth values on the extreme left column mean, respectively, that the truth values to which they are attached are designated (or true), antidesignated (or false), and designated and antidesignated at the same time. It is assumed here -and it will be argued later Chapter 6, sections 6c and 6d- that all values different from 0 are designated, and that all values other than 1 are antidesignated.

I think the distinction between the two negations is not just a logician's invention, but it is grounded on our way of talking (and ultimately, on there being degrees of non being). There are indeed degrees of negation. To say that 'there is no soap' is compatible with there being a tiny remaining of soap bar, which is not too efficient for washing hands, for example. But only when that left over portion is consumed, we can say 'there is no soap at all'. Again, sometimes it happens that at the moment we want to pay the bill in a supermarket, we realize that we have not brought any bank notes, and truly utter 'I have no money', although I may carry a few coins in my pocket. But if I am literally penniless, then the stronger negation is justified: 'I do not have any money at all'. One thing is to simply deny something, quite another to reject it. One rejects something only when the over-negation is involved. A flat or point-blank refusal is stronger than a mere denial. The question of whether there are semantically different negations is another side of the question of whether there are degrees of truth and degrees of falsehood. In this connection, we refer the reader again to Chapter 6, section 6c, where we will present an argument in favour of gradual truth.

As a result of the previous distinction, we must neatly set apart two kinds of contradiction. Over-contradictions, or super-contradictions, "$p\wedge\neg p$", "p and not p at all", are always totally false, irrational, never acceptable, etc. In contrast, simple contradictions, "$p\wedge\sim p$", "p and not p", are at least 50% false, but not necessarily absurd; indeed, some simple contradictions are partially true, but never more than 50% true. Consequently, among the formulas no longer tautological for the weak negation, '$\sim$', is the *Cornubia Principle*, or *ex contradictione quodlibet*, "$p\wedge\sim p \supset q$", the failure of which constitutes the defining feature of *paraconsistent logics*. And the disjunctive syllogism rule for weak negation also fails. But the strong negation counterparts of that principle and this rule continue to be valid.

Tautology?	$\sim$	$\neg$
$N0 \leftrightarrow 1$	✓	✓
$N1 \leftrightarrow 0$	✓	✓
$p\vee Np$	✓	✓
$N(p\wedge Np)$	✓	✓
$p \equiv NNp$	✓	✓
$p\rightarrow q \supset. Nq\rightarrow Np$	✓	✓

Tautology?	$\sim$	$\neg$
$p\wedge Np \supset q$	✗	✓
$p\vee q\wedge Np \supset q$	✗	✓
$p\supset q \supset. Nq\supset Np$	✗	✓
$Np \leftrightarrow NLp$	✗	✓
$p \leftrightarrow NNP$	✓	✗
$\frac{1}{2} \rightarrow. p\vee Np$	✓	✗
$p\vee q \leftrightarrow N(Np\wedge Nq)$	✓	✗
$p\wedge q \leftrightarrow N(Np\vee Nq)$	✓	✗
$N\frac{1}{2} \leftrightarrow \frac{1}{2}$	✓	✗

The previous table shows what principles are valid –in *Aj*– for each negation, where the 'N' should be uniformly replaced by either '$\sim$' or '$\neg$'.

To avoid ambiguity or misunderstanding, any absence of an intensifying expression should suffice to give you an indication that the negation involved is the weak one. In order to make reference to strong negation, explicit use of an intensifier is indispensable. Nonetheless, the mere 'not' in a classicist context should be interpreted as over-negation.

In the next section, I formulate some principles with weak negation. The reader who does not accept the particular distinction among two sorts of negations here advanced, is kindly requested to keep it in mind, since to substitute the strong negation for the weak one will result in a non-intended meaning, or perhaps in complete falsehood.

Third, concerning the conjunction and the disjunction, they take the minimum and the maximum values, respectively, out of the values of their members. That is,

$$/p\wedge q/ = \min (/p/, /q/);$$
$$/p\vee q/ = \max (/p/, /q/).$$

Fourth, we need to set apart two kinds of conditionals and, correspondingly, two biconditionals. The symbols '⊃', '≡' will represent the mere *conditional* and *biconditional*, respectively, both having the same characteristics as their classical counterparts. "p⊃q" is read as: "if p, then q", "p only if q". It is defined as "¬p∨q", by means of the strong negation. And 'p≡q' is the mutual entailment. That is, "p≡q" is defined as "p⊃q ∧. q⊃p". It is read as 'p is true if and only if q is true', 'p and q entail each other'. The truth tables of both '⊃', '≡' are indicated below.

On the other hand, the symbols '→', '↔' designate, respectively, the *implication* and the strict *equivalence*. Thus, "p→q" means that "p implies q", and "p↔q", "p is equivalent to q". As expected, equivalence is defined by means of double implication, this being a functor which compares the level of truth of antecedent and consequent. So, the truth value of "p→q" is designated or true (more specifically, ½) if the degree of "q" is more than or equal to that of "p"; it is 0, otherwise.

$$/p\to q/ = \begin{cases} \tfrac{1}{2}, \text{ if } /q/ \geq /p/ \\ 0, \text{ otherwise.} \end{cases}$$

Hence, other reading of "p→q" is that "q is at least as true as p", or "p is at most as true as q". And consequently, "p↔q" says that "p has exactly the same truth value as q", "p is as true as q".

Notice finally that the implication is stronger than the conditional, in the sense that the truth of "p→q" entails that of "p⊃q", but not vice versa: the truth of "p→q" does not follow from that of "p⊃q". And similarly, "p↔q" is stronger than "p≡q".

The truth tables beneath indicate the values of the functors just introduced, for a pentavalent logic.

⊃	1	¾	½	¼	0
1	1	¾	½	¼	0
¾	1	¾	½	¼	0
½	1	¾	½	¼	0
¼	1	¾	½	¼	0
0	1	1	1	1	1

→	1	¾	½	¼	0
1	½	0	0	0	0
¾	½	½	0	0	0
½	½	½	½	0	0
¼	½	½	½	½	0
0	½	½	½	½	½

≡	1	¾	½	¼	0
1	1	¾	½	¼	0
¾	¾	¾	½	¼	0
½	½	½	½	¼	0
¼	¼	¼	¼	¼	0
0	0	0	0	0	1

↔	1	¾	½	¼	0
1	½	0	0	0	0
¾	0	½	0	0	0
½	0	0	½	0	0
¼	0	0	0	½	0
0	0	0	0	0	½

On the other hand, the notion of *validity* employed here is a generalization of the standard one, which says that an argument is valid whenever it is truth preserving, that is, when the truth of the premises is not lost in the conclusion. In a many valued framework, we slightly colour this definition by adding two nuances: supposing that the argument premises are true, to some extent or other, it cannot be that its conclusion is completely false. In most cases, the value of the conclusion is equal to, or greater than the value of the least true premise, or even greater than the values of all premises. The only case I can think of where the value of the conclusion diminishes below that of the premises is in the case of the rule of acquiescence: Lp ⊢ p, where the premise can be totally true but the conclusion, only infinitesimally true. But this is alright; there is nothing logically wrong here.

According to this definition of validity, arguments are valid or not. However, we could admit that some inferences have more proving power than others. There are degrees of provability. Those arguments whose conclusion cannot be less true than the least true premise are more convincing than those in which the truth value of the conclusion can go below that of any of the premises.

To request that a valid argument preserve definite truth (Cfr. Burgess 1998: 247) would be too demanding. For one thing, in *Aj*, implications and equivalences, if they are designated, they take value '½'. So, this stronger definition would rule out arguments whose premises are implications or equivalences. We need to relax that maximalist requirement.

1b.- Different Versions of the Principles of Excluded Middle and of Non Contradiction

Once several functors of affirmation and negation are in place, we can discern various versions of the traditional principles of excluded middle (PEM, for short), and of non contradiction (PNC). The next table shows the most important schemes:

	PEM	PNC
Simple, or Weak	(1) p∨∼p (2) Lp∨¬p	(6) ∼(p∧∼p) (7) ¬(Lp∧¬p)
Strong	(3) p∨¬p	(8) ¬(p∧¬p)
Absolute	(4) H(p∨∼p) (5) Hp∨¬p	(9) H∼(p∧∼p) (10) ¬(p∧∼p)

(1) and (6), in contradistinction to (3) and (8), are called weak and strong, respectively, due to the kind of negation involved. But in another sense, (2) and (7) deserve to be named 'weak' for they are the least controversial. In fact, (2) affirms that "p" is true to some extent, or else it is totally false, while (7) asserts that it absolutely cannot be the case that "p" is more or less true as long as it is entirely false. Version (8) could also belong to this category of weak principles, because it states that any super-contradiction is completely false, which is something obvious. All these weak and strong principles are true.

To the contrary, all four absolute versions are plainly false. (1) and (6) differ from (4) and (9) in that the latter result from prefixing the over-affirmation functor to the former. Thus, (4) says that the simple PEM is totally true, whereas (9) says the same thing with respect to the simple PNC. (5) says that a sentence "p" is either totally true or completely false. (5) is deduced from (4) by first distributing 'H' over the disjunction, since the functor 'H' is truth-functional, and then by applying the replacement of equivalents, for "H∼p" amounts to "¬p": that "it is entirely true that not p" is exactly the same as that "p is completely false". For this same reason, (10) follows from (9).

Notice, finally, the contrast between (6) and (10): the former holds that a simple contradiction is false, but the latter contends that it is totally false. Only (10) excludes contradictions altogether, but not (6), which is compatible with the existence of simple contradictions. We can have both at the same time: p∧∼p and ∼(p∧∼p). This is just a contradiction of a second order.

To check the different valuations taken by the mentioned formulations, let me display the truth tables for the PEM in its several versions, in a penta-valent logic.

(1)	(2)	(3)	(4)	(5)
p ∨ ~p	Lp ∨ ¬p	p ∨ ¬p	H(p∨~p)	Hp ∨ ¬p
+1 1 0	1 1 0	1 1 0	1 1	1 1 0
±¾ ¾ ¼	1 1 0	¾ ¾ 0	0 ¾	0 0 0
±½ ½ ½	1 1 0	½ ½ 0	0 ½	0 0 0
±¼ ¾ ¾	1 1 0	¼ ¼ 0	0 ¾	0 0 0
-0 1 1	0 1 1	0 1 1	1 1	0 1 1

We can look at how true each version is. (2) is absolutely true; it is the less contentious. On the other hand, (1) is at least 50% true, or at most 50% false, whereas (3) can diminish below ½ its degree of truth, which can go as low as the lowest designated degree of "p". In flat opposition, versions (4) and (5) are not tautologies[2] at all. And we can see that their pattern is similar: they both are completely false whenever "p" takes an intermediate value. We can call (5) the *Principle of Exclusion of Intermediate Situations* (PEIS). We can see that fuzzy sentences, those which are neither 1 nor 0, imply the complete falsehood of the absolute versions of the PEM.

Parallel semantical considerations apply to the PNC. (7) and (8) are totally true. (6) is at least 50% true, or never more than 50% false. But (9) and (10) are wholly false, whenever there are true contradictions.

1c.- Two Formulations of the Principle of Bivalence

The Principle of Bivalence (PB, from now on) can be understood in two senses. Strictly speaking, it affirms that there are exactly two truth values: the Truth, and the Falsehood -symbolized as 'T' or '1', and 'F' or '0', respectively- which are jointly exhaustive, and mutually exclusive. By the first requirement it is meant that a sentence must have at least one of them; and by the second, that it has at most one of them. Thus, "p" is either T or F, but not both. In this strong sense, (PB) cannot be respected by any paraconsistent or many-valued logic.

There is another absolute version of the (PB), which is deduced from the absolute version of the PEM, the formula number (5) of the previous section: either "p" is totally true or it is completely false. But the sentence "p" is totally true, or totally false when the truth values '1' and '0', respectively, are assigned to it. Therefore, what the (PB) says in this strict formulation is that either $/p/ = 1$ or $/p/ = 0$. Expressed with the help of the 'definitely' operator[3], 'Δ', this strong version of the (PB) says that either "p" is definitely true or it is definitely false.

On the other hand, (PB) may be taken in a loose sense, without the requisite of mutual exclusivity, i.e., as the mere demand that the set of truth values be divided in two exhaustive subsets: the designated or true values, and the antidesignated or false values. In this wide sense, few many-valued and paraconsistent systems can still keep the (PB), despite their allowing non classical truth values. "p" is true or false, with lower case 't' and 'f'.

1d.- Vagueness or Fuzziness?

Before proceeding, let me indicate why I am not comfortable with using the word 'vagueness' to refer to the topic under discussion. As the *Oxford English Dictionary* attests, the *ordinary sense* of the adjective 'vague' turns around the notion of indefiniteness. Examples of vague items are: statements «deficient in details», «not precise or exact» language, or ideas «lacking in definiteness». One of the meanings of the expression 'in the vague' is «uncertain». Again, the *Merriam-Webster Third New International Dictionary, Unabridged* follows suit: 'vague' means: «stated in general... terms», «not having an exact or precise meaning», «not clearly defined», «not sharply outlined». According to this latter dictionary, one of the acceptations of the Latin etymological root 'vagus' is «uncertain». So, 'vague' as ordinarily understood has a negative connotation; a vague expression is defective in some way or other, usually not

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[2] A tautology is any formula that only takes designated values independently of the truth values of its component subformulas.

[3] This functor 'definitely' has been interpreted in several ways. "Δp" has been taken to mean either that "p is true in every precisification" by supervaluationists, or as meaning that "p is clearly true" by agnosticists, and finally in the sense that "p is completely true" by many-valued logicians. Actually, there is a difference between "Δp" and "Hp". 'Definitely' is a pragmatic notion, meaning something like that "p is beyond doubt", or " p is unquestionably true", while the use of 'totally' or 'completely' has the function of expressing in the object language that the sentence that it is attached to has the maximum degree of truth, 1. The functor 'H' designates the highest quantity in the scale of truth.

providing enough information which is wanted in the circumstances. 'Vagueness' is a pragmatic notion. But this lack of details has nothing to do with the phenomenon we are going to survey. Besides, 'vague' as explained above fits better with an indeterminist or agnosticist position rather than with a contradictorial gradualism, which I am going to defend.

And if we turn to the *philosophical sense* of the word, we are going to encounter a disparate array of definitions, so dissimilar as there are different conceptions on the matter, covering the whole spectrum of alternatives. The following characteristics have been put forward as essential to vagueness. It is unclarity (Williamson 1994b: 2), uncertainty (Channell: 20), indeterminacy (Field 1998: 200), a one-many relation of the representing to the represented (Russell: 66-67), boundarylessness (Sainsbury 1991a: 6), lack of precise boundaries (Tye 1994a: 281), tolerance (Graff 2002c: 54), possibility of borderline cases (Burns 1991: 3), ambiguity (Fine: 82), a source of incoherence (Read 1995: 176), gradualness (Dubois, Ostasiewicz, and Prade: 27), etc. Nonetheless, for each of the traits just mentioned, there is someone who has voiced a diametrically opposed opinion: vagueness is more over-determinacy than under-determinacy (Van Kerkhove 2003: 265), it is not a lack of sharp boundaries (Wright 2003c: 98), it is not constituted by tolerance (Greenough: 272), it is not defined in terms of borderline cases (Sainsbury 1991a: 9), it is not ambiguity (Channell: 34-5), it is not incoherent (Thorpe: 413), degrees are not distinctive of vagueness (Paoli 2003b: 381), etc.

Thus, there is wide discrepancy among philosophers over what we should understand by 'vagueness'. Therefore, it is of no help at all to appeal to "the" technical meaning of the word, because it is not unique, or one over which there is universal consensus. As it so often happens in philosophy, there is no general agreement, alas, not even in the initial or minimal characterization of the term delimiting the field of the discussion!

These are the reasons why, rather than using 'vague' or 'vagueness', I prefer the adjective 'fuzzy', and the substantive 'fuzziness', in their technical meaning to refer to the graduality in the possession of a property. A related sense is acknowledged by the *Oxford English Dictionary* when it records one of the acceptations of 'fuzzy' as it is used in computing or logic to designate a set which is «defined so as to allow for... gradations of membership». I am far from pretending that this word 'fuzziness' is neutral. But I shall claim that the phenomenon associated with the soritical series is nothing but fuzziness and is rather alien to vagueness, as ordinarily understood.

1e.- Semantics, Pragmatics, Realism and Pragmatism. General Orientation of the Present Work

In this section I will clarify that the overall trend of my work is a vigorously realist one, in opposition to a broadly subjectivist perspective. I will contrast both outlooks by presenting their antagonistic views about key semantic concepts, like meaning and truth, and about vagueness and the sorites. It will result that pragmatic considerations will be kept to a minimum.

Let me start by saying that the study of language can be approached from at least two angles, according to whether one emphasizes its connection to reality, or to the talking subject. My realist leanings inclines me toward explaining linguistic phenomena in terms of objects and their properties, in disagreement with the recourse to the human mind and behaviour, typical of other currents. The contrast has been aptly put by Michael Luntley (207), who affirms that physicalism attempts a worldly account of language, whereas certain idealism aims at a linguistic account of the world. These two general orientations are pervasive, surfacing in the characterizations of many -if not all- notions, as we will see throughout this section.

Roughly speaking, *semantics* will be understood here as the scientific inquiry about the relation between language and the world, or between an expression and an entity. More specifically, there are two semantic notions that receive focal attention: meaning and truth. Semantics is expected to make it clear what they are and what their nature is. We will mention some concrete conflicting proposals subsequently. However, not all philosophers have seen matters this way. For instance, from an anti-realist point of view, Michael Dummett thinks that semantics should explain our linguistic competence and understanding. For him, the notion of evidence is central to the meaning of a word, and so, the reference to the subject is implicit in his position.

On the other hand, the branch of *pragmatics*, as assumed in the present monograph, studies the relation between language and its users. In part, it is concerned with what speakers do with language (Glanzberg 2003: 185).

Now, *pragmatism* will be widely interpreted as the school of thought supporting the view that questions of language should be resolved by appealing to the role of the active subject, who strives to live in harmony with her environment (Hookway: 66). As illustrations of this approach, consider the following. B. S. Gillon (392) maintains that linguistic theory studies the human capacity to use language. Again, Neil Cooper (245) holds that the purpose of a theory of vagueness is to explain our practices. And when one has to decide between opposing conceptions on a particular issue by following a methodological criterion, some philosophers propose the canon that the only relevant thing is the attitude of users (Vieru), or that the issue be settled in favour of the position that fits the actual practice better (Sorensen 1991b: 79-80, 99). Nonetheless, concerning this question of how to settle semantic disputes, I personally subscribe the methodological rule recommended by Michael Devitt (1996a: 83-4) that one should give priority to ontological considerations; we shall put metaphysics first.

1e.i.- Pragmatist and Realist Views on Meaning and Truth

To further shed light on the different weights granted to reality or to the subject, let us see how meaning and truth are treated, first from the pragmatist point of view. According to a celebrated precept, meaning is use, or more broadly, it supervenes on the thoughts and practices of users. Language has meaning only by virtue of its use (Channell: 29). The meaning of expressions is determined by their use. This position is taken by Stewart Shapiro (*Vagueness in Context*: 5-6), who supports the belief that the judgements of speakers are part of the meanings of words. In the same spirit, Luntley (210) declares that an

> answer to the question 'How do names refer?' must illuminate what it is to think about objects.

These opinions lead to the suggestion that meaning is not objective. Meaning do not exist independently of our socio-linguistic practices (Goldstein, Unpublished: 6). According to behaviourism, meaning is determined by the observable behaviour of the speaker; however, in the opinion of Michael Devitt (1996a: 66), this position leads to the indeterminacy of meaning and to anti-realism.

An additional element in pragmatic accounts of meaning is the crucial function played by the context. It is often said that, in order for an expression to be meaningful, the context of utterance must be made explicit. The meaning of an expression depends on its context, and the degree to which this happens is its *indexicality* (Kempton: 40, n. 4). Thus, Scott Soames contends that: «To say that vague predicates are context sensitive is to say that they are indexical», such as the words 'I', 'here', 'now' (quoted by Jason Stanley: 270).

As for truth, a non-realist theory has it that mind-independent, objectively existing facts have nothing to do with true sentences. Terence Horgan has maintained the thesis that even though there are no vague objects nor vague properties, sentences about them can be true, because truth is not a direct language - world relation (1998b: 319-20, 327, n. 6). If this is so, then it is possible for an assertion to be correct and yet not to be required by the facts (Sainsbury 1992: 188). Indeed, pragmatist John Dewey held that the grounds of truth turn on our way of thinking. In the same vein, Donald Davidson has defended that there are no grounds of truth irrespective of our methods of verification. And Michael Dummett has championed the idea that what truth values there are and how they are assigned to their bearers is fixed by the purposes of the assertion (Glanzberg 2003: 165). Notwithstanding, there is a more ambitious project within this general tendency, seeking to carry the dependency on the subject even to the extreme of making the orderly world a mere product of humans. For example, Mark Heller sustains that persistent conditions and essential properties we attribute to things are the product of our conceptions (1990: 69). Again, famously Nelson Goodman has promoted the thesis that:

> The English language makes [objects] white just by applying the term 'white' to them (cited by Devitt 1996a: 103).

More recently, anti-realist Stewart Shapiro (37, 40) has pledged that:

> object *a* is *F* if and only if competent subjects judge it so.

I.e., it is the judgment of subjects what determines not only the truth of the statement '*a* is *F*', but also the being *F* of *a*. This is *idealism*.

On the opposite side of the debate, the one giving preeminence to being and reality, the role played by the subject, her judgements and interests, the context, and so on is reduced, to make room for more objective parameters. Thus, it is the link to the world what makes language meaningful (Devitt 1996a: 14). An expression has meaning insofar as it is associated with a real entity; so, a name stands for an object; a predicate, for a universal; and a sentence, for a fact. Thus, in a correspondence theory of truth, states of affairs are the meaning of sentences (David, § 3). The meaning of a word is what it stands for in the world. The *direct reference theory of meaning* advocates that the meaning of a name is its reference. A name refers to its bearer, independently of its sense or connotation, or of any descriptive phrase, and of its use in sentences. Meaning is determined by what in the external world is the source of the occurrence of the term (Wheeler 1975: 368). If this is truly the case, then the way how the subject reacts to a linguistic expression has no part in its semantic content.

And the same centrality of being is encountered in realist conceptions of truth. So, the sentence 'the cat is on the mat' describes reality as being some way (Glanzberg 2003: 164). To assign truth conditions to a sentence, which is part of the job of semantics, is to specify what the world must be like in order for the sentence to be true (Rayo, § 5.2). A realist account of the truth conditions of a sentence will reserve no prominent place for the subject, or the context. Thus, the *principle of transcendence* states that:

> a sentence may be true even if it is unknown (Young, § 1).

This has been argued by Timothy Williamson, who also contends that truth is not the same as the consensus of people. A sentence is not true due to its being accorded universal consent, for whole societies can be mistaken. Agreement among subjects does not imply truth. Universal consent is neither necessary nor a sufficient condition for truth. But Williamson does not go further. Yet, for a realist theory of truth, what is really a necessary and sufficient condition for truth is something ontological, namely, the existence of a mind-independent fact (in Kirkham: 73). Bertrand Russell, the first Ludwig Wittgenstein, George Moore, and more recently, D. M. Armstrong, have all supported the *truth maker principle*:

> for every contingent truth, there is something in the world, an existent state of affairs that makes the sentence true.

The ontological ground for the truth of '*a* is *F*' is *a*'s being *F*. Facts are the truth makers (Hochberg, 12-3; Armstrong 1997: 116; and 1989: 88). We will see later that this principle has been erroneously challenged by Willard van Orman Quine, Timothy Williamson, Roy Sorensen and Crispin Wright. On my part, I endorse the opinion that the facts which a sentence is about determine its (definite) truth (Cfr. McGee and McLaughlin 1994: 246, n. 17).

On the other hand, on our perspective, the context will be appealed to only in cases where it is necessary. On this point, I subscribe a *minimalist* position:

> a pragmatic, contextual aspect of meaning should not be considered part of what is said *praeter necessitatem*. In other words, it is only in case of necessity that we must incorporate something contextual into what is said (in Recanati: 255)

We will minimalize contextuality. Context dependency will be introduced only as a last resort (Kennedy 2003b, *in fine*).

We saw before that for an anti-realist position, it is the judgement of subjects what determines whether *a* is *F*. However, this appears to reverse the natural causal relation between judgement and being. Is the fact that *a* is *F* explained by the circumstance that the subjects judge that *a* is *F*, or vice versa, subjects judge so because the object *a* has the property *F*? (Eklund 2002a: 329). We think that a satisfactory explanation of why people confidently apply a certain term '*F*' to an object *a* is that *a* itself has some features that qualify it to be called an *F* (Rolf 1981: 81-2). If to be an *F* is to have the qualities *A*, *B*, *C*, *D*, and *E*, then the more present -or to a greater degree- these qualities are in *a*, the more confidently will the subjects describe it as *F* (Hampton 2000b, § 3; Hospers: 123). It seems plausible to hold that there is an ontological account of our linguistic practice, namely, the use of a word '*F*' by speakers is explained by the object's being a good exemplar of *F*. It seems to me that an object *a* is *F* not because we apply the predicate '*F*' to it, nor by virtue of any response given by the community of speakers, but because *a per se* instantiates, or partakes in the being of the universal *F*. It is the universal that gives particulars their nature (Armstrong 1989: 76-7, 94).

1e.ii.- Pragmatist and Realist Views on Vagueness and the Sorites

Finally, I will expound how these two antagonistic approaches, the realist and the subjectivist, yield divergent visions of vagueness and the sorites. Let me begin with the latter kind of trend. Again, what I want to stress is the fundamental function awarded to the subject among authors of this school.

Thus, ancient stoics contended that the problem of distinguishing truth from falsehood is within us, but not within the things as they are (Leib: 155). Similarly, Bart Van Kerkhove stresses that pragmatic factors -the relation between language and users rather than the relation between word and object- are the key to understanding vagueness and the paradox (2003: 252); and that the reason to favour a pragmatist account is our inability to draw boundaries (2000-2001: § 5). In general, it is voiced that vagueness has its root in our inability to make firm predications (Goldstein 1988: 450). In a borderline case, no definite, immediate answer is forthcoming (Needle, Ch. 1, § 1.1); and that speakers can go either way (Shapiro: vi, 10, 12; Walton: 228). This leeway in the standards of use with respect to a borderline case is clearly expressed by Delia Graff: it is up to me whether I say red or orange (2002c: 56-7). Vagueness has to do with the degree to which people categorize (Hampton 2000b: § 3), and that it is part of the way we deal with the world (Shapiro: 49). A borderline case does not have a semantic nor an ontological status, but it consists in a failure of judgement (Wright 2003c: 94).

And, concerning the sorites, it has been said that the reasoning has to do with observational judgements made by speakers (Goldstein 2000: 71). It is worthwhile noticing that, from a position making the world dependent on our concepts, it has been acknowledged that to deprive the major premise of truth commits the theorist to a sharply differential treatment of adjacent members in the soritical series without a sharply differentiating fact (Sainsbury 1994: 79; 1992: 182).

Among the pragmatist philosophers, I single out Douglas Walton, as a typical representative of pragmatism, who makes his case in a compelling way. He insists that his account of the slippery slope -one variety of which is the sorites- is pragmatic because it stresses the form in which it is used, and the context in which it is presented. The argument presupposes that there is a proponent of a thesis, and a respondent, who are engaged in a constructive dialogue. The slippery slope is used as a rebuttal of a contemplated course of action, by pointing out to the possible undesirable consequences that may follow if the first step is taken. Usually, the context is that of practical deliberation, or problem solving, for example, an ethical discussion about torture, or when the parties to the dispute try to decide on a social policy concerning euthanasia or abortion, etc.

What is most distinctive of Walton's pragmatist proposal is the way in which the slippery slope is evaluated. First of all, the formal validity of the argument is not an issue (90). What is correct (successful) or not is its use. An argument is *used correctly* if it advances the goals of a rational dialogue, «when it contributes cooperatively to the goals of an interactive critical discussion» (239); and it is used fallaciously only if it hinders those goals. The slippery slope is not intrinsically fallacious. The fallacy does not consist in a context free semantic failure, but in that it is used aggressively, when it is pressed too hard, in a way that prevents further questioning, giving the appearance that the debate is closed, that the foreseen consequences will inevitable ensue, when that may not be the case. So, the failure is pragmatic; the argument may be reasonable, but it is wrongly used. Most of the times, the slippery slope is used correctly, but sometimes it is not.

One point deserving attention concerns the treatment of the major premise of the sorites. In this regard, Walton holds that one has the «freedom» (67) to draw a precise line between a pair of objects whose difference is insignificant. That is, one has the «right» to declare that a_{i+1} is not F despite one's previous commitment to the F of a_i. He labels an 'inconsistency' any violation of the fairness rule, of treating like cases alike (132). He alleges that there is «reasonable arbitrariness» (60) in this «circumstantial» (139) inconsistency, since precise judgements are legitimate, necessary and useful for certain purposes (61). This strategy is akin to that vindicated by Nicholas Rescher: the abandonment of the weakest link in an inconsistent set is not absolute, but contextual, given certain purposes of a particular enterprise. The major premise of the sorites is «contextually untenable» (Rescher 2001: 52, 277-9; 80).

However, I believe there are more realist alternatives to these pragmatist approaches. Vagueness is not merely a matter of language or of our representations, but a feature of objects and of the world. (To continue the debate, see § 3a below, in this same chapter.) Vagueness in language is explained by vagueness in the world. The vagueness of a name is derived from the fuzziness of its referent (Rolf 1981: 85). An expression is vague just in case its referent is vague (in Eklund 2005: 34). Vague words stand for vague entities. A borderline sentence represents the way matters stand with respect to some feature of reality (King: 21). We believe that, only by not separating the fuzziness of language from the fuzziness in the world, one can avoid one of the inconvenient corollaries that idealism is committed to, namely, that our language is inadequate to faithfully describe reality as it is in itself (Cfr. Rescher 1958: 245). The world is fuzzy if and only if fuzziness is not a superficial phenomenon (Cfr. Peacocke: 133).

In summary, all the examples adduced show that a pragmatist perspective on linguistic matters in general, or on vagueness and the sorites in particular, attach great importance to the subject as the prime factor in the explanation of the phenomena. It is my conviction that a position giving priority to being and reality is at least as respectable as a pragmatist one, specially when one considers the alternative in the light of a paraconsistent background. Perhaps the problems levelled against a naive realist position, or against the direct reference theory of meaning, can be adequately responded if we entertain the possibility of rationally accepting contradictions. But of course, this is not the place to adjudicate the dispute.

What I do indicate is that, according to my point of view, one of the problems of fuzziness is to assign a truth value to a fuzzy sentence, or to determine whether an object has a fuzzy property or not (and to what degree). While the former question is semantic, the latter is ontological. In any case, we are not asking whether a fuzzy sentence is assertable in a given communicational context. What minimal threshold of truth is demanded from a sentence to be properly utterable in a certain situation is a pragmatic question. Should the sentence be 51% true, or 60% true, or...? I do not know the answer, and I am not sure whether somebody can know. But what I do know is that to require that a sentence be definitely true, or totally true, will be excessive, so much so that, perhaps, we would be prevented from uttering the bast majority of statements in our ordinary life. Conditions of assertibility are one thing, and conditions of truth are another. We will be concerned only with the latter. Moreover, it may be that, from a pragmatic perspective, the semantic status of a sentence is irrelevant to the appropriateness of its utterance. A falsehood may be legitimately asserted in certain context, and there are circumstances in which a complete truth is not utterable. So, assertibility conditions are more demanding; it may be that the absolute truth of a sentence is not sufficient for its being uttered. One thing is what the assertibility threshold is, quite another how things stand in reality.

On the other hand, it seems that though the context cannot change the truth value of the sentence, it can change its communicational relevance, its pragmatic pertinence.

In conclusion, there will be little space in this dissertation for pragmatical considerations.

2.- What Are the Problems?

We are going to canvass opposing approaches to two closely related issues: fuzziness and the sorites paradox. We want to know which theory among the alternatives is the most appropriate, or the least problematic. But before delving into the various proposals in the following chapters, we need to have a first look at each topic to be discussed. These are the subject matter of the next four sections.

3.- The Problems of Fuzziness

3a.- The Bearer of Fuzziness

A first point of marked disagreement concerns the subject or bearer of fuzziness. In the next sub-section we will examine in detail what fuzziness is. Here, for present purposes, let us understand fuzziness as the opposite of precision and exactness. When it is denied that the reality is fuzzy, usually what is meant is that objects and properties are sharply bounded, that they have crisp borders, finely carved, not blurry, coarse, or badly defined. On the other hand, a precise word has definite and neat conditions of application.

Once we have a rough and ready initial characterization, we ask what fuzziness is attributed to. Two different perspectives can be distinguished. First, the ontological view, which holds that fuzziness is primarily a feature of items figuring in an ontological inventory of reality, such as objects, properties, relations and facts; they all can be fuzzy. If this is so, then the world itself is fuzzy inasmuch as it contains at least one fuzzy entity. Language also is fuzzy but derivatively. Second, the semantical view, which maintains that fuzziness is a quality only of linguistic expressions, specifically of a sentence or, alternatively, of its components, like nouns and predicates. What is fuzzy is not reality itself, but our description of it by means of language. This stand could be extended to include our mental representation in thought: concepts or judgements can also be fuzzy. Since both parties agree that language is fuzzy, the debate turns around the issue of whether fuzziness is a real characteristic of some kind of entity. So, we are confronted with a choice: must fuzziness be taken as something objective, as an intrinsic constituent of the nature of certain things existing in the universe, or should it be reduced to something subjective, somehow dependent on our language, on our conceptual scheme, or on limitations of our perceptive capacities?

If, for a moment, we restrict the discussion to the case of monadic predicates and properties, then the question is: are there fuzzy properties? And this is an ontological query. Nihilists, agnosticists, supervaluationists, pragmatists answer no, while fuzzy and many-valued logicians say yes. We could express the problem in semantical terms: what is the meaning of a fuzzy predicate '*F*'? What does it refer to? And the alternatives are: we can assign to '*F*' either a classical set or a fuzzy set.

Of course, the topic is not easy at all. Maybe, at the end, the issue is whether existence is fuzzy. And to positively hold that being is fuzzy might be seen as extremely revisionary, or excessively revolutionary. Thus, one can understand the strong antagonism to real fuzziness. It is astonishing nonetheless that one of the most rigorous thinkers on the subject, Timothy Williamson (2003: §§ 5, 9) resists the following inferences from (1) to (2), and from (1) to (3):

(1) $\sim\Delta Ba \wedge \sim\Delta\sim Ba$
(2) $\exists F \exists x\ (\sim\Delta Fx \wedge \sim\Delta\sim Fx)$
(3) $\exists p\ (\sim\Delta p \wedge \sim\Delta\sim p)$

Let us suppose that sentence (1) is a case of fuzziness: Alfred is neither definitely bald, nor definitely fails to be bald. From this, it would not follow that there is at least one property and at least one entity such that it is fuzzy whether x has *F*[4]. It seems at first sight that the charge of non sequitur is surprising. Sentence (3) affirms that there is at least one fuzzy fact, one which neither definitely obtains nor definitely fails to obtain. It states the thesis that reality is fuzzy. But again (3) would not follow from (1). I do not doubt that a semantics can be devised so that both entailments get frustrated. That is what happens in supervaluationism and agnosticism. The existence of any borderline case does not make reality fuzzy.

Both agnosticists and supervaluationists assign a classic set to a fuzzy predicate. Then, from this perspective, it is natural to maintain that linguistic fuzziness does not have to rely on ontic fuzziness, or that we can perfectly use a fuzzy language in a non fuzzy world (Keefe 2000: 15). However, I just want to record that this is striking. If fuzziness is a trait assigned only to assertions but not to reality, then a mismatch is introduced between the fuzzy language and the exact and precise world. Yet, this consequence already is an inconvenience for a realist who wants to defend a relation between language and the world as direct as possible.

Be that as it may, how can we adjudicate the matter of whether there is real fuzziness in the world? If a fuzzy ontology has any chance of making a compelling case in its favour, then there must be certain phenomena which cannot be satisfactorily explained except by postulating fuzzy sets. What facts can we understand better thanks to fuzzy sets? The reasons to introduce fuzzy sets are the following (Bouchon-Meunier, 1995: 9, 162). Fuzzy sets are required because they allow a progressive passage from a property to its opposite, say, from black to white, avoiding abrupt, sudden transitions, and the imposition of arbitrary cuts. And again, elements may belong to a fuzzy set in a measure less than absolute, so that they are authorized not to belong completely to a set nor to its complement; hence, fuzzy sets permit their members partial membership. For example, a dark grey patch may belong to a high degree to the class of black things and to a low degree to the class of white things. The main idea behind fuzzy sets is that the more an object approaches the typical characterization

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[4] According to Williamson, a detailed explanation of the fallacy makes reference to the illegitimate reversal of the scopes of the vagueness operator 'neither definitely... nor definitely...' and the definite description 'the fact that Alfred is bald'.

of a class, the stronger its membership to the class should be. Thus, fuzzy sets are in a better position to treat "badly" defined concepts (like 'centre of the city', 'old'), or categories that are not well separated and that partially overlap with each other, intermediate situations (almost black), or approximate values (around 2 kilometres).

So, there is a range of affairs whose occurrence provide evidence for the hypothesis that there are fuzzy sets, which will be the ontological correlates of fuzzy predicates. We claim that all the mentioned facts are nothing but facets of the existence of graduality in nature. Therefore, the whole issue of whether fuzziness is real or not hinges on whether reality is gradual or not. If the world is gradual, then we need fuzzy sets, and fuzziness will not be merely linguistic or mental. Fuzzy language reflects a fuzzy reality. If there are no degrees of a property, then supervaluationists, agnosticists and contextualists are correct in believing that properties have sharp limits, and perhaps we will be better off dispensing with fuzzy sets.

If philosophers have avoided to impute fuzziness to the real world, that, I conjecture, is due to a fear of ascribing contradictions to it. We will see that this misgiving is not grounded.

3b.- The Nature of Fuzziness

Concerning the nature of fuzziness, there is also a lack of unanimity as to how to characterize it. It is one of those philosophical concepts about which schools are divided as to which essential features should be included in its definition, each trend having its own preferences. But several attempts have been made to identify the peculiarity of the phenomenon. I examine those features that have been proposed as being more prone to be accepted by all sides taking part in the debate: indeterminacy, borderline cases, lack of borders, and sorites susceptibility.

3b.i) Indeterminacy?

A fuzzy fact is constituted by an object which neither possesses nor lacks a property. Think, for example, of a glass which is half filled with water. It is neither full nor empty, supposing one quality is the negation of the other.

(1) $\exists x(\sim Fx \wedge\sim \sim Fx)$.

If we have to classify a fuzzy object, x, as either *F* or not-*F*, x defies a neat classification, since it does not properly fall in either category. Applying De Morgan to (1), we get:

(2) $\exists x\sim(Fx \vee \sim Fx)$

the simple negation of the weak principle of excluded middle. And in this sense, a fuzzy situation weakly falsifies the PEM, and therefore it is a kind of soft indeterminacy. Yet, this is not the same as saying that the simple PEM fails, for it remains true to some degree. Nonetheless, one aspect of fuzziness is that it does entail the simple negation of the PEM. And vice versa, the ontological principle of excluded middle:

(3) $\forall x(Fx \vee \sim Fx)$

denies the occurrence of fuzzy entities, since it entails that there cannot be any object x such that it is neither *F* nor not-*F*:

(4) $\sim\exists x(\sim Fx \wedge\sim \sim Fx)$.

This is precisely the third possibility that is excluded.

Thus, fuzziness and the simple PEM weakly negate each other. We see that (2) denies (3), and (4) denies (1). These two claims of mild incompatibility are correct, as long as we keep the simple negation. Notice, however, that in a paraconsistent system as Aj, we can have the four statements, from (1) to (4), all asserted as partially true, and therefore also false to some extent.

Moreover, when we introduce the operator 'definitely', symbolized by 'Δ', we gain further results. The failure to comfortably cataloguing a fuzzy object x happens whenever x is neither definitely *F* nor definitely not-*F*. Then, translating this into logical notation, a fuzzy fact is:

(5) $\sim\Delta Fx \wedge\sim \Delta\sim Fx$.

And, by De Morgan, an instance of fuzziness, as in (5), amounts to:

(6) $\sim(\Delta Fx \vee \Delta\sim Fx)$.

If we take (5) or (6) by themselves, as statements of an isolated characteristic of fuzziness, they are alright. In fact, rendering (5) in terms of the over-affirmation functor 'H', as in many-valued logics, a fuzzy situation is:

(7) $\sim HFx \wedge\sim H\sim Fx$

which is the same as

(8) $\sim HFx \wedge\sim \neg Fx$

i.e., a fuzzy object neither perfectly possesses a property nor utterly fails to possess it. All these assertions, from (5) to (8), are acceptable. Indeed, (5) or (8) may be seen as grounds for (1). But remember the difference between 'Δ' and 'H'. See footnote 3 above.

Nevertheless, if it is claimed that (6) conflicts with the principle of excluded middle, then there is a real problem, because the central idea behind the previous attempt to characterize fuzziness by means of the negation of the simple PEM was that the fuzzy object resisted to be classified into either of two exhaustive alternatives. Notice that this is correctly expressed by (1) or (2), but not by (5) nor by (6). Certainly, none of the latter two statements denies two contradictories; indeed, "$\Delta\sim Fx$" is not the negation of "ΔFx". In order for (6) to be the denial of the weak PEM, the scope of the negation in the right disjunct should encompass the definitely functor too and not merely the predicative sentence.

One who affirms that (6) denies the principle of excluded middle is immediately committed to take this principle as:

(9) $\Delta Fx \vee \Delta\sim Fx$

i.e., either x is definitely *F* or it is definitely not-*F*. But this is an absolutist rendering of the PEM, that must be rejected, if there are fuzzy objects or properties. In a many-valued logic, (9) could be understood as:

(10) $HFx \vee H\sim Fx$

which is equivalent to:

(11) $HFx \vee \neg Fx$

i.e., either x is completely *F* or it is not-*F* at all, which is the predicative counterpart of formula (5) of section 1b above, the Principle of Exclusion of Intermediate Situations. This is an all or nothing dilemma. (9) and (11) tell us that the two mentioned radical alternatives exhaust the range of possibilities. From a semantical point of view, what the PEIS tells us is that a sentence is either totally true or entirely false. And, finally, if we undo the distribution of the affirmation functor 'H' over the disjunction in (10), we obtain:

(12) H ($Fx \vee \sim Fx$)

which affirms that the simple principle of excluded middle is completely true, which is the predicative form of scheme (4) of section 1b.

Yet this is a mistake. If there are fuzzy situations, and we see no reason to deny this, then the simple PEM must be false, up to a point. More accurately, the PEM is false in exactly the same extent as there are fuzzy facts. And vice versa, these are unreal to the degree that the weak PEM is true. We saw this before.

But the incompatibility between (6) and (9) is of another sort, it is downright. Because of the presence of 'Δ', both claims can only receive a classical truth value. And, since one is the direct negation of the other, they are contradictory: there is no way at all that both can be true simultaneously. If (9) is a law of logic, then there cannot be any fuzziness at all in reality. And if (6) correctly represents what a fuzzy fact is, then (9) cannot be a law of logic. Hence we have to decide between them. I keep (6) and reject all the formulations from (9) to (12). Principles like (9) or (11) declare as possible only the extreme cases, excluding all intermediary situations. We would be left with a desert world, containing very few things: those that paradigmatically possess, or fail to possess a given property. All fuzziness would be gone.

Concerning the question of whether the fuzzy object, x, is utterly indeterminate, we answer negatively, since that could only happen if x utterly did not possess any of the opposite properties, *F* or not-*F*, in any degree whatsoever. But I do not see how this could happen, given the ***Principle of Inverse Co-variance of Opposites***:

(ICO) the more an object is *F*, the less non-*F* it is, and vice versa.

For example, the more the door is open, the less it is closed. There cannot be any strong indeterminacy with respect to *F* if we are able to place the purportedly indeterminate object in a series of elements ordered by the relation of 'being more *F* than'. In this series, as we move from one extreme to the other, we know that each consecutive element is less *F* in the same extent as it is more not-*F*. If there are degrees (which will be established in Chapter 4, § 1a, and Chapter 6, § 6b-c), then we can escape the false alternative presented by the Principle of Exclusion of Intermediate Situations.

In conclusion, we can say that a fuzzy object is weakly indeterminate, in the sense that it neither is *F* nor not-*F*, because it is not completely either. Therefore, the simple principle of excluded middle is false, not entirely, but partially, though it is also partially true. Moreover, we have refused to acknowledge the absolute versions of the PEM. It is wholly false that only what is definitely so and so has a place in the ontology. It is absolutely essential to include a third option beside the extremes.

Notwithstanding, this first approximation to fuzziness, as weakly indeterminate, is purely negative: it only tells us what a fuzzy object is not. Fuzziness is a sort of *tertium quid* with respect to the opposites, neither the one nor the other. But exactly what is it? Eliminating certain possibilities may help us to begin understanding what fuzziness is, but something else must be added; a positive characterization is wanted, if we are to make real progress.

3b.ii) Borderline Cases

A second attempt to characterize fuzziness appeals to the notion of a borderline case. Originally, this notion is intended as an explanation of semantic fuzziness: an expression is fuzzy if it has the possibility of borderline cases. For example, x is a borderline case of a predicate '*F*' when x neither is within the extension of *F* nor outside of it, but x -as it were- sits exactly on the dividing limit. That is, it is presupposed beforehand that the space is split in two regions, one belonging to the field of application of the expression in question, and the other beyond its range, and that there is a line effecting the partition. Further, the location of the fuzzy case is initially indeterminate, in the sense that it is neither in nor out, but immediately after it is specified that its proper place is precisely on the boundary.

Now, let us try to give the idea an ontological twist. The ontological space of properties is demarcated by frontiers separating each quality from the rest. Imagine the enclosed area of property *F*. What is outside the border is not-*F*. Hence, an object is fuzzy if it does not lie on either side of the limit but stays right on it.

Have we made some progress with respect to the initial characterization of fuzziness? At first sight, yes. It seems we have advanced one step, since we have assigned a place to the fuzzy object, instead of depriving it of being somewhere on the map. Still, the nature of the borderline case is not wholly elucidated. It remains to be seen whether it continues being neutral or if it belongs to both sides. Is it like staying on the bridge over a river dividing two countries, and therefore in nobody's land, or more like being in the dusk or dawn? If the notion of borderline case is going to be of any help, it is necessary to further explore its nature to have clarity about its status with respect to the two opposites flanking it. Otherwise, it seems we have only found a name for the problem we had before.

Is the borderline case indeterminate? It does not seem to be so. Remember our first characterization of fuzzy objects: x neither possesses nor fails to possess property *F*.

(1) $\sim Fx \wedge \sim \sim Fx$

We only need to suppress the double negation in the right conjunct to reach our solution:

(13) $\sim Fx \wedge Fx$

the borderline case is and is not *F*; it possesses both opposites, and therefore is contradictory. An indeterminate situation is a contradiction in disguise. If dusk is neither day nor night, then it is both, but each only to a certain extent. Dusk is day for there is still some intensity of dim light brightening the sky, although the sun may have just disappeared behind the horizon. And it is night since darkness is almost everywhere, but not quite.

The example of how the day ends and the night begins is a nice illustration of a gradual process. We may say that dusk sets in, if not before, when the lower arc of the solar sphere begins to disappear behind the horizon line. At this moment the luminosity of the sky starts diminishing, though perhaps we do not notice. It may take about half an hour until the upper arc of the sun is no longer visible. At that time, the western part of the sky is still light blue although on the eastern part it is dark blue, but nonetheless blue, not black. Meanwhile, the moon and Venus have shown up, and later on the rest of the starts.

If there are contradictory entities, that is so only because the object possesses both opposites in a limited measure. A many-valued logic or a fuzzy set theory can explain why contradictions arise: they are made possible only by degrees in the possession of the opposites.

To try to avoid the passage from (1) to (13) by invalidating double negation will be a highly *ad hoc* move.

If we are disposed to make room for a third category of borderline cases in addition to the extremes, it is better to talk of a borderline zone, for, in most typical cases, not all objects falling in it will be equally distanced from the poles: some will be closer to, others, farther away from one of the extremes. So, in general, this region is not homogeneous, but it is internally differentiated. If we consider the segment of the spectrum of colours going from yellow to red, we can discriminate there more than one shade of orange. Some bands will be reddish orange, while others will be yellowish orange. This example also serves to illustrate the fact that this middle area consists of an overlap of opposites. Orange has as many nuances as there are ways of mixing the proportions of each of its ingredient colours, red and yellow.

Summarizing this second point, we can say that most standard cases of fuzziness demand the recognition of a third intermediary thick zone of variegated borderline cases as a welcome and necessary addendum to the logical landscape. Yet the status of the fuzzy object as a borderline case is not (strongly) indeterminate but contradictory.

To end this topic, we should address the question of the limits of this zone of intermediate cases. Does it have precise borders or not? If it itself is imprecisely delimited, then the scope of the borderline cases is fuzzy. Authors talk here of *second order vagueness* or *higher order vagueness*. If borderline objects were indefinite, being in a third category other than *F* and not-*F*, and the set they form were itself indefinite, in the sense that there were some objects which neither belonged nor failed to belong to this third category, then it would be vague which objects are vague.

Personally, I think that, in the case of a bounded property, as explained in the next section, there are clear limits to which objects are fuzzy and which are not. Fuzziness begins the moment we depart from the total possession or lack of a property. And these two conditions are exactly determined. Any object which neither completely has a property *F* nor altogether lacks *F* qualifies as a fuzzy object. If we had an ordered series of objects such that the first, a_0, is completely *F*, the last, a_z, is utterly not-*F*, and the intermediate members diminish their degree of possession of *F* so that there is a gradual transition from the first to the last, then except both, a_0 and a_z, all the rest are fuzzy. So, there is no indefiniteness nor unclarity concerning which entities are fuzzy. The borderline cases are definitely circumscribed.

Nonetheless, there is a sense in which the range of fuzzy objects may be said to be fuzzy. This will be explained in Chapter 6, dealing with the gradualist approach. See § 6a.

3b.iii) No Boundaries?

Let us now review an alleged third mark of fuzziness. It has been claimed that a fuzzy object or property is one which does not have limits. An illustration of this is the property of being tall as applied to human beings: there is no precise minimum height for persons to qualify as being tall. The same lack of cutoff points would affect colours. If we take the spectrum of colours, where exactly does red begin and end? Again, how many hairs does a man need to have in order to be bald? How much money must a person possess to be rich? It seems there is no accurate, natural answer to these questions, even if all the contextual factors are laid down. And, consequently, it is concluded that there are no borders. Other properties and objects satisfying this condition are: young, cold, fast, big, heavy, far, heap, tadpole, chair, etc. So, first we are asked where we should draw the line that marks the end of a property and the beginning of its opposite; and the answer is expected to be sharp. But from our apparent impossibility to solve the question, it is deduced that there is no border limiting the property. Notice that the rejection of frontiers is not restricted only to a certain kind of them, but it is general. It is not that fuzzy entities lack precise, clear-cut borders, but possess fuzzy borders; rather, they are boundaryless.

Of the three features that we have seen so far, this is the one which has the least number of supporters. However, first, we must concede that the question concerning the place where the exact borderline should be drawn is sensible, and that it admits of a variety of reasonable answers. For example, agnosticists declare that there is one exact limit though it is unknowable to us, whereas supervaluationists maintain that there are several precise demarcations possible, all equally legitimate. Furthermore, we also have to admit that the existence of continuity in nature makes it very difficult for us to set limits to fuzzy properties, because a border of a quality divides the entities into those that possess it from those that do not. And it seems that we do not find these borders anywhere. They are not apparent.

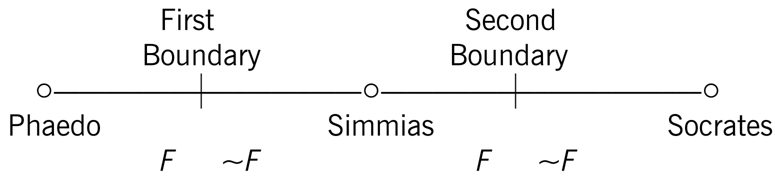
Despite these two concessions, we have reservations about the conception of fuzzy entities as those that are boundaryless. One reason for this is that we believe that a fuzzy property has many borders. In order to show this, consider two things, x and y, which are both *F*, but x is *F* to a degree less than y. For example, x is less tall than y, though x is also tall. My claim is that there is a limit whenever there is, between two things, a relation of inferiority in the possession of a property. Why? Because of the already mentioned *Principle of Inverse Co-variance of Opposites*:

(ICO) x is less *F* than y in the same extent as x possesses more of the opposite non-*F* than y.

That Simmias is less tall than Phaedo is equivalent to the fact that he is shorter than Phaedo. And, if Simmias is shorter than Phaedo, then he is short, in virtue of the *Aristotelian Rule for Comparisons*:

(ARC) nothing can have a property in a greater or lesser degree if it does not possess it unqualifiedly.

Otherwise, how could something have less or more of a property if it does not possess it at all? But to be short is not to be tall. Hence, Simmias is not tall. Consequently, to have less of a property is not to have it. Therefore, when we pass from y to x, we pass from *F* to non-*F*; that is, there is one limit between the two. The property 'tall' has a first frontier here. And this case can be reiterated many times, as many as there are pairs of individuals that can bear the relation of inferiority. So, if Socrates is less tall than Simmias, by the (ICO) Principle, he is shorter than Simmias, and then, by the (ARC), Socrates is short, and not tall. This means that the property 'tall' has a second limit between Simmias and Socrates. And so on. Thus, fuzzy expressions are multiply bordered. The existence of degrees entails the existence of boundaries.



But remark that the kind of limit proposed here is not of the old type, which partitions the universe into two mutually exclusive and disjoint classes. Rather, it is a paraconsistent border, one which allows things not possessing the property to be in the same class as those possessing it. The situation is a little more complicated than in the traditional picture. Let us return to the previous example. We saw that Simmias falls on the negative side of the first limit: he is not tall. However, Simmias is taller than Socrates; therefore, he is tall, in view of the (ARC). And because Socrates is short, a second limit occurs between them. Relatively to this second boundary, Simmias is tall; he can be grouped together with Phaedo, from whom he was separated by the first limit. The fact that Simmias is not tall makes the first boundary what it is: something that cuts off; but, since Simmias is also tall, the first boundary does not discriminate: there are tall people on both sides of the fence. This is why the boundaries are not rigid, or crisp, but allow an overlap of opposites. The frontiers are relaxed, permitting migration in both directions, and border crossings. The limits divide and do not divide.

We conclude, fuzzy entities do have soft borders.

Two last observations should be made before we leave this issue. First, if a logical system $\underline{S}$ does not have degrees of truth, as it is the case in CL, it cannot express neither weak denial nor partial affirmation. That is, if the meaning of '$\sim$' is given by its truth table, then, in the absence of non standard truth values, one cannot specify the values taken by "$\sim$p". And again, in $\underline{S}$, there is no way to express that a sentence "p" is more or less true but not completely true (Lp $\wedge$ $\sim$Hp). In a classical framework, if "p" is true, it is totally true. That entails that the notion of border employed by one who lacks gradualist resources, has to be maximalist, to wit: a boundary divides entities that are completely so-and-so from those which are utterly not so-and-so: Hp on one side, ¬p on the other. More generally, the divide would be between what is definitely *F*, from what is definitely not-*F*: Δp | Δ∼p. This is a *sharp boundary* (Wright 1994: 142; Keefe 2000: 28). By contrast, in a gradualist framework, the *boundary* is understood simply as the difference between what is *F* and what is not-*F*: p | ∼p.

Second, there is another sense in which properties may be bounded or not. Let me introduce two definitions, which are important because they give rise to different treatments of the sorites.

A property *F* is *bounded*, or closed on both sides, iff there is an object, a_0, which definitely possesses it, and another, a_z, which definitely lacks it; a_0 is properly called a paradigm, prototype, or exemplar of *F*; a_z may be called the perfect not-*F* instance; it is a paradigm of the opposite of *F*. An illustration of this first type of property is the quality of being tall, as it applies to the adult human population currently living. Extreme cases of a_0 and a_z are, respectively, the tallest and the shortest men in the world.

F is *unbounded* or open-ended iff it lacks either a_0 or a_z. This implies that there is an infinite number of objects which either increase their possession of *F*, each object successively exemplifying *F* in an ever higher degree without reaching 1, or decrease their degree of possession of *F*, each object successively exemplifying *F* in an ever smaller degree without reaching 0. An example of this second kind of property which lacks a_z is the quality of being close to the Eiffel Tower. As points augment their distance from the tower, they decrease their degree of possession of the property of being close to the tower. But there is no point that is 0% close to it, since there are always more points which can still be farther away.

3b.iv) Sorites Susceptibility

A fourth and last trait of fuzziness which is likely to receive a majority approval, though not unanimity, is that a fuzzy situation leads to the sorites paradox, i.e., it is responsible for the generation of absurdities. This fourth aspect of fuzziness is dynamic, having to do with small changes in the circumstances. As an illustration, consider the temporal stages of Mary, as measured by the beats of her heart, and compare any two contiguous phases. Then, if we are going to judge Mary as young or not, it is obvious that she will deserve the same treatment at both adjacent stages, in view of the almost perfect identity between both phases. Said otherwise, it does not happen that she is young at stage *i*, but not so at the next stage, *i+1*. This is the major premise of the argument. If we grant this much, and begin with Mary at the age of 12 years old, when she is really young, then, by repeated application of the major premise, we are going to extend the attribution of the quality young throughout Mary's life until she is very old. But, by hypothesis, she is not young at all when she is -say- 95 years old. However, by force of the argument, we conclude that, even at that advanced age, she is still young, which is absurd. Fuzziness thus is charged with being a source of incoherence.

We certainly think it is an essential feature of fuzziness its being the cause of the sorites paradox. What we dispute is that it is incoherent, in the sense that it is absurd. But this is already the topic of paragraph 4. So, we abstain from making further commentaries until then.

Recapitulating the four points made so far, we recall that a fuzzy object has been characterized as being neither completely *F* nor completely not-*F*; that therefore the introduction of a borderline zone is mandatory; that a fuzzy predicate has many bounds; and that fuzziness is what gives rise to the sorites.

3c.- The Alethic Status of Fuzzy Sentences

The previous discussion has focussed on fuzziness of a fact: it is fuzzy whether an object possesses a property. But we can approach the issue of fuzziness from a semantical point of view, considering what the truth value of a fuzzy sentence is. For example, let 'p' be the sentence 'The cup of coffee is cold', when the coffee is lukewarm. On this respect, there are several ways of looking at the question.

To begin with, classical logic, by its embracing the Principle of Bivalence, dictates that 'p' is either T or F, but not both. Thus, indeterminist or contradictorial interpretations of fuzzy sentences are excluded. Remark that (PB) does not necessitate that one know which truth value 'p' has. As a matter of fact, 'p' would be either 1 or 0, but it would be impossible for us to discover which is the case. This is the position of agnosticist philosophers, like Roy Sorensen and Timothy Williamson.

Other authors of an indeterminist conviction, supervaluationists among them, believe that 'p' is neither true nor false, lacking any alethic status whatsoever.

For many-valued logics, 'p' is neither 1 nor 0, but has a non classical truth value, though there is a difference of opinions among partisans of these logics concerning the designation of the new truth values. When a fuzzy sentence takes a third value, ½, most authors interpret it as indeterminate, neither designated nor antidesignated, whereas some proponents of fuzzy logics supporting the allegiance to the Principle of Bivalence loosely construed permit the positive designation of ½, as well as the introduction of infinite degrees of truth, all of them designated.

And lastly, paraconsistent positions uphold the inconsistent status of 'p': it is both true and false.

So, the options concerning the truth value of 'p' are the following: 1) 'p' has exactly one of the classical two truth values; 2) it does not have any truth value; 3) it has a non classical truth value; 4) it has a truth value that is both designated and antidesignated. It is clear that our stand on this particular issue is the third and the fourth.

4.- The Sorites Paradox

Let me begin by clarifying the meaning of the two words appearing on the title of this section. First, 'sorites' is an ancient Greek word signifying a heaper, or accumulator, one who adds things. The root from which the word is derived is 'soros', which means a heap, from where the alternative name of the argument comes, namely, 'the heap paradox'. This denomination refers not only to the specific subject matter of the reasoning, but also to its logical structure, since it consists of an accumulation of premises, as we will see immediately.

And second, 'paradox' commonly denotes an argument which has apparently true premises, and is apparently valid[5], but has a contradictory or absurd conclusion. Consequently, if we want to avoid the conclusion, there are basically two ways out (if we discard the possibility of invalidating the argument by the mere presence of fuzzy words in it): either at least one premise is faulty, or the rule of inference used is invalid. There are also philosophers who take the argument as really paradoxical: the premises are certainly true, and the form of the argument is genuinely valid, and yet the conclusion is thoroughly false. That would be for them a proof that fuzziness is irremediably illogical, and that therefore there could not be any logic of fuzziness. This would be outside the realm of logic. We will examine this position in detail later in Chapter 5, § 2.

4a.- Origins

As for the origin of the paradox, it is only known that its inventor was Eubulides of Megara, a contemporary of Aristotle, in the third century BC. The initial form of the argument consisted in a series of questions: is one many?, is two many?, is three many?, and so on. The context of later discussions in times of the ancient stoicism was a particular aspect of the problem

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5 A valid argument, let us recall, is one that, having the premises true to some degree, it is impossible for the conclusion to be absolutely false.

of induction: how many experiences are sufficiently many before we can safely generalize a law in medicine? What is outside our ken is what Eubulides' purposes were. Perhaps the sorites was not merely an intellectual curiosity, a puzzle interesting in itself, but was used as a weapon against a target in a polemic. I conjecture that Eubulides had affinity with the monistic thinking of Parmenides and Zeno of Elea.

4b.- Informal Structure of the Argument

The best known intuitive forms of the sorites are the following two.

i) The Paradox of the Heap (the sorites *sensu stricto*)

First premise:	Zero grains are not a heap.
Major premise:	The addition of a single grain cannot make the difference between what is not a heap and what is a heap.
Conclusion:	Nothing is a heap.

ii) The Paradox of the Bald Person (phalakros):

First premise:	A hairy person is not bald.
Major premise:	The loss of a single hair cannot turn a hairy man bald.
Conclusion:	Nobody is bald.

The structure of both arguments is the same. The first premise consists of an object a_0 which paradigmatically, undoubtedly, determinately, possesses the property *F*. If we were going to construct a sorites for the colour red, then a ripe tomato could serve as our standard of red object. For 'tall', as referring to the height of humans now living, we will have to begin with the tallest man in the world. And so on. If we want to start safely, then the best way is to mention prototypes of *F*. Agnosticists will advise us to use unquestionable or evident cases. So, this premise is the hardest to question. Yet we will see that even this evident premise has been disputed by nihilists.

The major premise asserts the fact that a minimal difference, scarcely observable, in some underlying dimension, *G*, on which *F* supervenes, does not justify a change in the status of *F* concerning its being attributed to, or withheld from some object. The principle behind this will be labelled the *G-F Correspondence Principle*:

(*G-F* Cor.) no tiny alteration in *G* is capable of creating a significant change in *F*

It indirectly asserts the proportionality between variations in *G* and modifications in *F*. Thus, the removal of a grain from a heap is not going to cause its disappearance. Similarly, if I am hairy, the loss of a hair will not make me become bald. In other words, one minute fluctuation in the subvening property *G* does not drastically affect the membership of *a* to the set of things *F*. A very small variation in *G* cannot cause the transition from *F* to not-*F*, or vice versa. Thus, this major premise also seems obvious in itself. However, it has been challenged by many authors who do not have any other option, since they want to leave the logic of the argument intact.

On the other hand, the reasoning may take two forms, according to what kind of process the second premise involves: addition or substraction of units relevant for the application of the predicate, i.e., increase or decrease of the subvening parameter *G*.

The conclusion is arrived at by continuous reapplication of the process involved in the major premise. If at the beginning, Frank has 100.000 hairs on his scalp, after one hair is lost, he continues being hairy. And again he still is hairy with the loss of one more hair. At each step of alteration in *G*, there is no radical transformation in *F*. His condition of being hairy persists. But not for ever. At the end of the long process, after 100.000 stages, when he has lost his last hair, Frank is absolutely bald. Nonetheless, if the reasoning is sound, he must be hairy. And this is really preposterous.

Thus, something must be wrong. But what? Both premises clearly seem to be correct. This is the paradox: from seemingly true premises, we arrive at an absurd conclusion. This is our second big problem.

Notice that the conclusions of both the sorites and the phalakros generalize the status of the first object to all others. If the prototypical case a_0 is F, then all a_i are F; and conversely, if a_0 is not-F, then none of the a_i is F. Anyway, apparently, the effect of the major premise is to deny a transition from one opposite to the other. What it does is to extend the application or non-application of the property F from the paradigmatic object a_0 to all the remaining objects, up to a_z, if there is one. It spreads the fuzzy property, and in this sense we can say that what is fuzzy diffuses itself. Fuzziness is diffusive[6].

4c.- Formalization of the Argument

In order to analyse the validity of the argument, it is necessary to give it a logical form, so that the employed rule of inference becomes explicit. The alternatives depend on what formulation the major premise receives. Basically, two different forms will occupy our attention. If the major premise is a conditional, or an implication, then the reiterated rule is *modus ponens* (MP); but, if it is a disjunction plus weak negation, then the rule is disjunctive syllogism (DS).

Note that both forms presuppose that F is bounded: the a_z of the conclusion is not F at all; otherwise, the argument is not really paradoxical, the conclusion not being absurd, but merely contradictory. Of course, this assumption is not in force if the property F is open-ended. For a discussion of this last case, see two paragraphs below (sub-section 4e).

i) Conditional Major Premise

First Premise:	a_0 is F
Major Premise:	If a_i is F, then a_{i+1} is F
Conclusion:	a_z is F
Rule of inference:	MP: p, p⊃q ⊢ q

ii) Implicational Major Premise

First Premise:	a_0 is F
Major Premise:	That a_i is F implies that a_{i+1} is F
Conclusion:	a_z is F
Rule of inference:	MP: p, p→q ⊢ q

iii) Disjunctive Major Premise with Weak Negation

First Premise:	a_0 is F
Major Premise:	Either a_i is not F, or a_{i+1} is F
Conclusion:	a_z is F
Rule of inference:	DS: p, ∼p∨q ⊢ q

To be rigorous, a universal quantifier binding a_i and a_{i+1} should be prefixed to all three major premises. Accordingly, we should also have mentioned the rule of universal instantiation, which will allow us to obtain the required particular premises to apply MP and DS. But, since these complications will not affect the subsequent discussion, I have omitted them altogether.

6 In Latin, 'diffusus' is the past participle of 'diffundere', which means to spread out, to scatter.

Which form should we prefer? This question is fundamental because we will give the argument a different treatment partly depending on which symbolization we choose for the second premise. The full range of possibilities is open. If the major premises take a conditional or implicational form, then at least one of them is totally false, but the rule is valid. In this case, the argument is not sound. In contrast, if the major premises are disjunctive and use weak negation, then all of them are true, but the rule is invalid. And, on the other hand, when the property F is open-ended[7], all premises are true, and the conclusion too; hence, the rule is valid, the argument being sound. If the rule of inference is disjunctive syllogism, it will be *de facto* valid, because the conclusion is true, but the rule as such is formally invalid.

In order to support the option for the disjunction plus weak negation reading of the second premise, let us introduce the notion of a soritical series.

4d.- The Soritical Series

The Soritical Series underlies the construction of the sorites, and is an ordered collection of elements differing with respect to G. It is this base parameter G which orders the members. In the case of a bounded property, the soritical series has two extremes, each being an ideal instance of the opposites F and not-F, respectively; that is, it begins with a_0, which is definitely F, and ends with a_z, which is definitely not-F. In the case of unbounded properties, the soritical series has at most one extreme; at least on one side, there is no end to the series, which will extend *ad infinitum*. Either way, the central feature of the soritical series is that any two contiguous members, a_i and a_{i+1}, are subjectively indiscernible concerning their possession of F, because they barely differ relatively to G. Indeed, due to this very small variation in G, there is a correspondingly tiny, objective dissimilarity in F among the adjacent members, but it is not observable with the naked eye, it is below our unaided threshold of discrimination, so that, as a matter of fact, we cannot distinguish a_i from a_{i+1}. Thus, a_i and a_{i+1} are scarcely dissimilar, and therefore, predominantly similar.

It is not easy to avoid a contradictory description of this particular aspect of the soritical series. I believe that the series is indeed contradictory, so a contradiction must appear somewhere sooner or later. But I will not press this issue here.

Let me provide an example of a soritical series. It is a bounded one, for the property 'tall' restricted to our actual human world now. Imagine a sequence of persons ordered by height so that the difference in G between one subject and the next is so little a magnitude that it is imperceptible: one tenth of a millimetre. On one extreme stands the shortest person; on the other, the tallest. Under these conditions, if we are going to compare any two adjacent fellows, we will be entirely unable to detect any difference in tallness among them.

Now, there are at least four ways to capture in logical notation this almost complete similarity with minimal dissimilarity between two elements, a_i and a_{i+1}, which are next to each other. As before, the initial universal quantifier is dropped for the sake of simplicity.

(SP)	$Fa_i \wedge Fa_{i+1} \vee .\ {\sim}Fa_i \wedge {\sim}Fa_{i+1}$
(CP)	${\sim}(Fa_i \wedge {\sim}Fa_{i+1})$
(Par.P)	${\sim}Fa_i \vee Fa_{i+1}$
(Pre.P)	$Fa_i \supset Fa_{i+1}$

i) ***Similarity Principle*** (SP):

for any two adjacent members, either both are F or neither is.

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[7] Remember that, in the present context, a property F is open-ended when there is no last a_z which is a perfect non-F, but there is instead an infinite number of a_i which are F, each in a lesser degree than its predecessor.

This means that two subsequent members should be co-classified. Their likeness grounds the application of the property to both or its withholding from both. Both a_i and a_{i+1} fall on the same side of the boundary. This feature is closely connected with:

The Fairness Principle: Like cases must be treated alike.

Fairness is part of what justice means. (SP) is the embodiment of logical equity. The conception of fuzziness based on (SP) could thus adopt the slogan: "vagueness as fairness".

ii) ***Continuation Principle*** (CP):

> for any two contiguous elements, it cannot happen that only the former is *F* while the latter is not *F*.

That is, their strong resemblance prohibits any discrimination between them. With other words, (CP) tells us that the border between *F* and not-*F* is not between a_i and a_{i+1}. Again this negative principle can claim to be a facet of justice: it is indeed unfair to treat like cases in an unlike manner. It is felt that a petty difference in *G* among a_i and a_{i+1} is not enough to give them a contrary treatment.

iii) ***Parity Principle*** (Par.P):

> for any two subsequent members, either the first is not *F*, or the second is *F*.

iv) ***Preservation Principle*** (Pre.P):

> for any two flanking neighbours, if one is *F*, so is the other.

Observe that the difference between (iii) and (iv) is obliterated in CL, due to the absence of distinct negations. Recall that the conditional used in (Pre.P) is defined by means of strong negation: the truth value of "$p \supset q$" is equal to that of "$\neg p \vee q$". In contrast, the negation employed in (Par.P) is weak. So, (iii) and (iv) express quite diverse things.

The first three principles are not independent from each other. (SP) is the stronger of them, in the sense that (CP) and (Par.P) follow from it. These two last principles are logically equivalent, and for that reason, (Par.P) also joins its parent and twin brother in upholding the conception of fuzziness as fairness. The principle which is disconnected from the other three is the fourth one. There is indeed no way to deduce (Pre.P) from them, in view of the contrast between the two negations.

Given that (Pre.P) does not belong to the family, the real alternative is between it and (Par.P). At this moment, we have already all the background needed to answer our previous question: which formulation should we give to the major premise of the sorites? Conditional or disjunctive (with weak negation)? It is time now to make a selection.

Unfortunately for (Pre.P), there is a reason disqualifying it as a suitable representation of the relation among the members of a soritical series: not all of them respect (Pre.P), if the series is bounded. In effect, consider the last two elements of the series, say a_{z-1} and a_z. By hypothesis, a_z is not *F* at all, for the series is bounded. But a_{z-1} is *F*, to some degree however small. (This last claim can be supported as follows. We know that a_{z-1} and a_z are ordered by a relation of inferiority: a_{z-1} is less not-*F* than a_z. If that is so, a_{z-1} is more *F* than a_z, by the Principle of Inverse Co-variance of Opposites. And if a_{z-1} is more *F*, it is *F*, by the Aristotelian Rule for Comparatives). Therefore it results that the conditional 'a_{z-1} is $F \supset a_z$ is *F*' will be completely false, having a true antecedent but a totally false consequent. That means that the preservation condition is violated at the end of a bounded series.

The same conclusion unfavourable for classical logic would be reached if we replaced the strong negation for the weak one in the (SP), the (CP), or the (Par.P). That is, the following principles formulated in classical logic become entirely untrue for the last two members of a bounded series.

(SP)$_{CL}$ $Fa_i \wedge Fa_{i+1} \vee. \neg Fa_i \wedge \neg Fa_{i+1}$
(CP)$_{CL}$ $\neg(Fa_i \wedge \neg Fa_{i+1})$
(Par.P)$_{CL}$ $\neg Fa_i \vee Fa_{i+1}$

Take, for example, the continuation principle for the strong negation, $\neg(Fa_i \wedge \neg Fa_{i+1})$. If we suppose that $/Fa_i/ = 0.01$, while $/Fa_{i+1}/ = 0$, then the second strong negation will be 1, and hence, the conjunction must take the value of the left conjunct, which is 0.01. Therefore, the first over-negation shall be 0.

The conclusion we have reached is alarming for CL supporters. We have shown that CL is unable to logically capture the relation among members in a soritical series, since it cannot describe the series by any true principle. This puts CL in a disadvantageous position for it cannot but judge the soritical series as inexistent.

However no such breakdown affects any of the three original principles formulated with weak negation, all of which remain true throughout the series. So, we must render the major premise by means of a disjunction and weak negation. Then, the rule used is DS.

4e.- Sorites for an Unbounded Property
Beside the sorites using a bounded series, we need to survey another variety which uses an unbounded series, with no a_z. We will reserve the name '*slippery slope*' for this kind of reasoning. Let us imagine a collection of balls arranged in a straight line, each one in contact with the next, and let us call the first of them 'A'. The property, or relation, we are interested in is that of 'being close to A'. The hypothesis to be reduced to the absurdum is that there is a ball, Z, on the other extreme of the line, such that it is in no way close to A. We begin our reasoning with ball B, and unconditionally assert that it is close to A, since there is nothing which could be closer to A than what B is. If something is close to A, that is B. Is C close to A? We think it is, since the relation of closeness is taken to be transitive:

B is close to A
C is close to B

C is close to A.

The motivation for the transitivity of the closeness relation is that the distancing away from A by a few centimetres cannot bring about the end of the closeness to A. Analogously, not because I move myself one step away from the Eiffel Tower I stop being close to it, or position myself far from it. The same Principle (*G-F* Cor.) supporting all major premises of the sorites is operative here: a small difference in *G* cannot produce a substantial difference in *F*. Observe that this first subconclusion does not affirm that C is as close to A as B is. It only says that the relation still holds between A and C, but it does not mention the degree, which, of course, will diminish.

If the validity of the rule is granted, then we apply the same reasoning to the next ball: D is close to C, but C is close to A; therefore, D is also close to A. Again, bear in mind that D will be near to A, but still less than what C is; nonetheless, D is close to A. That is the conclusion. And we repeat the argument with respect to E, F and all subsequent balls. But this means that, by transitivity, Z too will be close to A. Yet, by hypothesis, Z was absolutely not close to A. Therefore, by *reductio ad absurdum*, there is no such point Z. Hence, every ball is close to A. By the way, the name 'slippery slope' suggests that once you begin to slide, you cannot stop half-way, but have to go all the way down, through an endless path.

In brief, the argument reveals the same logical structure as the ordinary sorites we have seen; in effect, it starts with the concession that the paragon case has the property *F*, and demonstrates that all other cases have also *F*. The difference is that we do not have an archetype of the opposite of *F*, but instead an infinite number of objects each of which exemplifies not-*F* in a greater degree than its predecessor but without ever reaching degree 1.

In this formulation of the slippery slope, we have a limited number of attitudes to take to face the reasoning: either we accept the conclusion, or we reject the rule of inference. For a discussion of these alternatives, please see section 6 below.

5.- Denials of the Major Premise

Let us see what would happen were we to reject (wholly deny) the truth of at least one major premise of the sorites argument for a bounded property. We have distinguished at least three variants of it, so the negation of the major premise also adopts various formats. Let us enumerate all rejections, on each of which we will comment immediately after. In the first place, informally, the rejection of the major premise would commit us to maintain that the loss of a single hair can turn a hairy man bald, or that the addition of a single grain can make the difference between what is not a heap and what is a heap. In the second place, remember that we identified the Principle (*G-F* Cor.) as the one offering the foundation for the major premise. Its refusal would mean that a tiny alteration in *G* is capable of creating a significant change in *F*. And in the third place, formally, if the laws pertaining to the negation of the quantifiers, and the De Morgan principles are in place, then the strong negation of two of the logical versions of the major premise, (CP) and (Par.P), is equivalent to:

(DT*) $$\exists a_i, a_{i+1}\,(\mathrm{H}Fa_i \wedge \neg Fa_{i+1})$$

i.e., there is a member in the series such that it is completely *F*, whereas its next neighbour is not-*F* at all, which we will call the '***Discontinuity Thesis***', and the stand supporting it '***Discontinuism***'. It then affirms that there is a sharp boundary somewhere in the series.

Concerning the Preservation Principle, $\forall a_i, a_{i+1}\,(Fa_i \supset Fa_{i+1})$, its absolute negation yields a slightly different result, namely, $\exists a_i, a_{i+1}\,(\mathrm{L}Fa_i \wedge \neg Fa_{i+1})$. What is interesting to note is that the absolute negation of the Similarity Principle, $\forall a_i, a_{i+1}\,(Fa_i{\wedge}Fa_{i+1} \vee.{\sim}Fa_i{\wedge}{\sim}Fa_{i+1})$, produces the existence of a pair of contiguous members, a_i, a_{i+1}, such that either at least one of them is absurd (completely true and completely false at the same time), or there is a sharp limit between them: while the one is totally *F*, the other is altogether not-*F*.

Now let us comment on each refusal of the major premise. First, it is incredible that the loss of one hair will turn a hairy man bald. Against this, we must say that it is empirically false; we observe the contrary. Every time I comb myself, I lose a few hairs, without thereby becoming bald. To truly go bald, I would have to loose thousands of hairs, but not only one.

The root of the mistake is the rejection of the Principle (*G-F* Cor.). To believe that there could be a mismatch of proportionality between variations in *G* and in *F* is to assimilate the passage from *F* to not-*F* to the collapse of an impressive playing-cards castle by the most gentle touch. That again is hard to admit. There should not be a wild discrepancy between changes in *G* and in *F*, because the correspondence between both changes as conveyed by the Principle (*G-F* Cor.) seems to faithfully reflect the common sense truisms that the higher a person is, the taller she is, the more money a person has, the richer she is, etc. Perhaps, part of the scruple against (*G-F* Cor.) is the existence of seeming counter-examples, like the fact that a few more cents in my purse do not make me richer, for I may be poor. However, this objection does not hold water, because it presupposes that nobody can be rich and poor at the same time. I surmise that once we remove the fear of contradictions, the misgiving against (*G-F* Cor.) vanishes thoroughly. Being hosted within a paraconsistent system, the (*G-F*

Cor.) Principle is immune to attacks of this sort. We find that it seems to be highly implausible that a significant difference in *F* can be caused by an insignificant difference in *G*.

Equally unacceptable is (DT*). It postulates the existence of a pair of members in the series such that a_i is the last to be totally *F*, while a_{i+1} is the first to be not-*F* at all. More exactly, it inserts a sharp divide between the two members. The first problem with this is its arbitrariness. Nothing in nature will justify the exact location of the boundary. Take the property of being an adult human being. What age will mark the beginning of adulthood? Assuredly, whatever exact age you pinpoint, it is not going to have a foundation in reality. Suppose your put the limit at 18 or 25 years old. Whichever the case, the question arises of why not one day before, or one day after. The drawing of the borderline wherever you favour to situate it utterly runs counter to the principle of sufficient reason. There is no ground to put the limit in a particular place rather than somewhere in the vicinity. The boundary can at best receive a practical apology, in virtue of its utility for certain purposes. And thus it appears to be a mere stipulation, without any real basis in the nature of things.

The second reason why a sharp boundary cannot be located between any contiguous members is their intimate likeness. It is because they resemble each other so much that a disparity between a_i and a_{i+1} of the sort postulated by the discontinuist is out of place. The sharp boundary places in antithetical categories two individuals that are indiscriminable. It is like inventing a dissimilitude where all the observational data point to the contrary. When you are backed up by a many-valued and paraconsistent system, you can introduce a dissimilarity where there is a similarity, because you have degrees and tolerate innocuous contradictions. But this is not the case with discontinuism, which imposes upon adjacent members a severance, exaggerating the weight of the divergence. It is true that a_i and a_{i+1} are a little bit unequal, but that is not enough to judge them hardly alike. The repudiation of degrees and contradictions forces on us a spurious dilemma.

Connected with this is the question of fairness. To discriminate between like cases is unjust. Like cases deserve equal treatment, unless there is a relevant reason against that. But a minimal discrepancy among contiguous members is not sufficient to segregate them.

Thus, we gather that the rejection of the major premise of the sorites for bounded properties is burdened with difficulties: it is empirically false, it goes against common sense truisms, and against the displayed likeness among neighbouring members, it is arbitrary and unfair. Therefore, it is advisable to keep the major premises, in accordance with fuzziness. If we do not accept the conclusion of the sorites, we should block the argument by means other than by giving up the premises. That is, we should entertain the strategy of invalidating the rule of inference. Indeed, disjunctive syllogism is invalid for the weak negation, though valid for the strong one. Fortunately, the alteration of CL as contemplated here will only bring gains, with no losses.

We have just argued in favour of maintaining the premises, and against their dismissal. Notwithstanding, this double declaration must be qualified. We do not advocate the total truth of any major premise. That means that they are to some degree false. We know that the paradox intends to prove, by the workings of gradual metamorphoses, the absence of a transition from one property to its opposite. If that is a sophism, we must allow that, by imperceptible transformations, we do go from *F* to not-*F*. It is to some extent true that:

$$\exists a_i, a_{i+1}(Fa_i \wedge \sim Fa_{i+1})$$

We accept this for it solely expresses the existence of a soft limit between a_i, a_{i+1}, due to the weak negation involved.

6.- The Refusal of the Slippery Slope

Let us explore now how we can react to the sorites for unbounded properties, or slippery slope, in the form given above in paragraph 4e. It is clear that to deny its second premise would destroy the first premise too, since both premises hold for exactly the same reason: each ball is close to the preceding one because the distance between them is null; they are side by side. In these circumstances, were we going to refuse the conclusion, we would rather have to deny the transitivity of the closeness relation.

So, let us suppose that, from the facts that B is close to A, and that C is close to B, we cannot infer that C is close to A, insofar as the relation of 'being close to' is not transitive. Unfortunately, this move is fraught with grave consequences, though at first sight it may appear a promising one. In fact, what follows from the rejection of the transitivity of the closeness relation is that, in order for an object to be close to A, it would need to be in contact with it, that is, completely close to it, as close as B is. Less than that would amount to a failure to being close. Indeed, a boundary has been drawn after ball B, so that it is the last one to be close to A, while C is the first to be not close. If the distance from A increases a little, going beyond that between A and B, then that is counted as no longer being close to A. We will call this position '*maximalism*'. In general, it affirms that:

in order for x to be *F*, it is necessary that x be absolutely *F*.

Thus, only the person who has zero hairs on her scalp is bald, and only Bill Gates is rich. To be cold is to have a temperature of zero degrees Kelvin. To be good is to be optimum. To have a property *F* is the prerogative of an elite possessing *F* at the superlative level. The first privileged members to pass the exacting test will be those included in the McGuiness' Book of World Records. Only the best examples will have the right to figure among the instances of each property. And this demand is extended to truth also:

Alethic Maximalism: a sentence is true only if it is totally true.

From this, the *Maximalization Rule* follows:

$$p \vdash Hp.$$

But this is an extremist position hard to be reconciled with. When the requirements of membership are lifted to the utmost degree, the net result of it is a massive dismissal of "presumed" members, even of those in good standing. Yet that is an unbearable impoverishment of reality. It will deprive us of all non prototypical examples of every property. All imperfect instantiations of *F* will not be regarded as genuinely belonging to the set of things *F*. Solely the genius would be intelligent. But, since some genius are less genius than others, just the most brilliant will qualify as intelligent, perhaps one alone.

In fact, this elimination of deficient instances is equivalent to the outright abolishment of fuzziness.

A second unacceptable consequence derived from maximalism is that there will not be degrees of anything different from the maximum. That an attribute comes in degrees means that an object can have it in varying extents, such as large, high, medium, low, slight, or in a greater or lesser extent. If maximalism were true, then nothing could have a property to a certain extent lower than the top degree. Only what is 100% *F* would be *F*. C could not be less close to A than what B is. There would not be such relation as 'less close to A'. C is purely and plainly not close to A, period. The absolute principle of excluded middle would be in force here: or fullness of being or complete non-being. There is no space for intermediate situations.

As a corollary, we would not be able to make comparatives. If there are no degrees, x could not be more or less *F* than any y. We could not say that Simmias was taller than Socrates, since Simmias, being shorter than Phaedo, would not be tall to begin with. If he is not tall in the first place, nor can he be taller. Only the tallest person in the world would be tall (if we consider this property as bounded), and the rest would be not tall. All this is unsatisfactory.

Additionally, other reason preventing the maximalist from being able to make comparatives is that the most direct, simple and illuminating way to understand a comparative, like x is more *F* than y, is to analyse it as a comparison of the respective degrees of possession of *F* by x and y, that is, the degree to which x is *F* is greater than the degree to which y is *F*. So, that Rome is warmer than Brussels means that Rome is warm to a greater degree than that to which Brussels is warm. This account is straightforward: the comparative construction 'is *F*er than' is explained in terms of degrees of *F*. x is redder than y iff x is more red than y. But if there are no degrees of any property, then we are deprived of the most obvious manner to clarify the comparatives.

Therefore, in view of the severe difficulties of maximalism, it is commendable to avoid it, and to admit the conclusion of the slippery slope. When a property *F* is unbounded, everything is *F*.

Yet, perhaps there are ways to avoid this conclusion evading maximalism too. A first such an attempt is that maybe the rule of inference, transitivity of closeness, has only *local*, but not global, *validity*, i.e., it is correct provided that solely a few number of applications are made. The problem would be that the rule is reiterated too many times. We should then restrict its use. Thus, we could deduce that C, D, E, F and possibly a few other balls are close to A, but then, at a given point, in order to check the diffusion of the attribute of being close to A, we do not authorize any more instantiations of the rule. Suppose we accept that I is close to A, but even if J is close to I, we block the inference that J is close to A, for transitivity does not have global validity. Thus, we would make room for some balls being close to A, and at the same time we would eschew sliding all the way down along the slippery slope.

The problems with this strategy are the same as those of discontinuism, because halting the validity of the rule after some initial applications issues in the introduction of a borderline between balls close to A and those not close to it. But this limit will be arbitrary, will fly in the face of the similarity between neighbouring balls, and will be unfair for the contiguous balls flanking the frontier.

Still the corollary of the impossibility of comparatives extracted from maximalism will find resistance. We saw that a natural way to elucidate the comparatives was the following:

x is more *F* than y = the degree of x's being *F* is greater than the degree of y's being *F*.

But there are alternatives to this gradualist conception. One of them is that instead of analysing the comparative *F*er in terms of the positive predicate *F*, we could take the opposite approach: that of reducing the positive form to the comparative. The meaning of vague predicates would be parasitic on comparatives. First we will have to indicate a minimal threshold for the possession of *F*, and then establish that in order for *a* to be *F*, it needs to reach at least that threshold. For example, to be tall *a* ought to measure at least 1.8 m., i.e., *a*'s height must be 1.8 m. or more. In general,

'*F*x' $=_{df}$ the degree of x's *G* is equal to, or greater than certain standard.

Thus, this scheme requires that the vague term be rendered precise, and so, it has the same effect than the tactic of limiting the validity of the rule of transitivity of the closeness relation, and therefore, it also shares the drawbacks of the latter.

We postpone a larger treatment of maximalism until Chapter 6, section 6d.

7.- The Tasks of a Theory of Fuzziness

We have had a first contact with the two problems of fuzziness and the sorites paradox, as well as with the main stands we are going to delve into for the rest of the work. Now we bring together the phenomena asking for an explanation, or the most important questions that have to be addressed by any theory of fuzziness.

1.- How is it possible to change from *F* to not-*F* by means of a soritical series?
2.- What is the nature of the transition from one opposite to the other?
3.- Why does the transition occur?

It is obvious that there is a variety of antagonistic answers, and our purpose is not merely to have a catalogue of possible responses, for we are seeking the truth. We want to know which the best available theory is. No doubt it will have problems. But it is hoped that they are not insurmountable, or that they are less serious than those affecting rival theories. In any case, we need to adjudicate between the contenders. To this end, I propose that the adequate theory is the one offering the best explanation to the problems previously mentioned. As Kirkham (19) affirms, the correct view is the most enlightening with respect to the problems.

8.- Degrees and Opposites in Ancient Philosophy

Now that we have seen the problems that we will be concerned with, and exposed some positions, I consider it is necessary to refer to a few pertinent ancient doctrines concerning being and non being, and the opposites in general, so that the contrast between contradictorial gradualism and its rival is contemplated in its historical roots. I will not go into exegetical analyses. My only purpose here is to briefly explore conceptions of Heraclitus, Parmenides, Anaxagoras, Plato and Aristotle, which, suitably interpreted, can be taken as the forerunners of contemporary positions. This I hope will serve to establish links between schools that share the same spirit, and bring into focus relevant ontological theories that are too easily forgotten in discussions of the subject.

8a.- Heraclitus and the Unity of Opposites

First of all, I would like to mention Heraclitus' view of the relation among the opposites. We discern at least two sorts of relations: (a) unity and sameness, and (b) harmony or agreement.

In fact, in some passages, he holds that the bond between the contraries is elevated to the level of unity or sameness. For instance, in his fragment:

> B57: ...day and night... are one,

and

> B60: The path up and down is one and the same.

B88 adds a gloss:

> ...the same thing is... living and dead, awake and asleep, young and old;
> for the latter change and are the former, and again the former change and
> are the latter (Barnes 1988: 103, 120).

Thus, we may contend that it is in the course of a change that the opposites get merged (*Id.* 1979/1: 72; Kirk: 109, 142-44, 152, 154). But in what sense are they one and the same? We may interpret Heraclitus as trying to give a picture diametrically opposed to a dualist vision: the progression from day to night is not without intermediary stages: there is a twilight zone (*Ib.*: 174, n. 1). From the moment the sun begins to set until it gets very dark, there is a period of decreasing brightness and increasing darkness (Kahn: 109-10). Dusk constitutes a seamless transition, where day and night are fused. Again, the opposites are not separated (Kirk, Raven and Schofield: 191, n. 1), but in a melange; instead of driving a wedge among the opposites, Heraclitus integrates or amalgamates them. As soon as one introduces something in common among the opposites, like a bridge closing a gap, they are –in a sense– no longer two, but one and the same (Kahn: 205). Insofar as they cannot be told apart, they are one.

Beside this strong link among the opposites, there is another kind of soft tie. Let us read Fragment 8, whose second part, though classified as genuine by Diels-Kranz, most probably is a paraphrasis of a truly Heraclitean idea (*Ib.*: 193; Kirk: 219-20):

> What is in opposition is in agreement, and the most beautiful harmony comes out of things in conflict... (Sweet: 5).

We can alternatively translate the verb of the opening sentence, «τὸ ἀντίξουν συμφέρον» as: «what is opposite coincides» (Pabón: 556, *sub voce*). This fragment clearly indicates that whatever tension there is among the opposites is only partial, that their contrariety is not absolute so that there is room for a compatibility between them, a "harmonious strife" (Barnes 1979/1: 80; Kirk: 205, 216-17, 402; Kahn: 197, 199-200, 204, 284; Guthrie 1971: 43-7; Vlastos 1955: 137; Stokes, in Graham: 4). That is, one opposite does not exclude the other: they can be present in the same subject at the same time (Vlastos 1955: 143; Plutarch, in Barnes 1979/1: 319, n. 24; Guthrie 1971: 436; Zeller, in Kahn: 323, n. 239), in the same respect. Precisely, the soritical series may be seen as a nice embodiment of this *coincidentia oppositorum*. The series may consist of 101 elements. Of these, the extensions of both opposites have an intersection of 99 members, from a_1 to a_{99}. But if two sets, X and Y, have an intersection to which at most one element in each set does not belong, this may be seen as ground to affirm that X and Y are (largely) coincident.

So, there is a way to construe Heraclitus' sayings that tries to respect the letter of his texts, acknowledging the contradictoriality of his thought but without charging him with incoherence or absurdity. For Heraclitus, the entities of this moving and impermanent world are contradictory; the contradictory properties can beautifully coexist in the same subject. Heraclitus brings together what for dualism stands worlds apart. The opposites coalesce.

8b.- Parmenides and the Principle of Exclusion of Intermediary Situations

Let us continue our historical overview by passing review to a conception of reality that is better known for its rejection of change, the thought of Parmenides. Perhaps his denial of movement is the logical result of his refusal of non being (Curd: 122-24), contradiction and ultimately of degrees of being. Be that as it may, the only aspect which I want to emphasize is his opposition to degrees and the immediate consequences of that. In fragment VIII of his Poem (Burnet: 187, [lines 106-8]; and 186, [lines 79-80]), he explicitly says:

> Lines 46-7: For there is nothing which is not that could keep it from reaching out equally, nor is it possible that there should be more of what is in this place and less in that...
>
> Lines 22-4: ...there is no more of it in one place than in another, to hinder it from holding together, nor less of it, but everything is full of what is.

Thus, Parmenides has rejected degrees of existence, or so he has been commonly interpreted (Guthrie 1969: 31-33, 43-46; Coxon: 203, 215-16). It is important to realize the grounds for his denial. One of the reasons mentioned is that there is no non-being inside being which could hinder what is from being homogeneous. And the other reason can be taken as more general, that there is nothing that could be an obstacle to the fullness of being.

Hence, if there are no degrees of being, the only alternatives remaining are the extremes, as stated in line 11: Therefore what is

> ...must... either be altogether or be not at all (Burnet: 186).

SNotice that the intensifying adverbs are in the original Greek. Perhaps we have here the very first formulation in the history of philosophy of the principle of excluded middle; but since the particular version given is a maximalist one, it is better to name it *Principle of Exclusion of Intermediary Situations*. Of course, the Parmenidean reasoning proceeds to show that the second disjunct in no way is. But what matters to us is that the alternatives are not simply to be or not to be, but the more radical to be *fully* or not to be *at all*. Undoubtedly, this is an all or nothing ontology, without more or less, and without non being. (Curd -pp. 5, 76-7, 81-2, 88, 93- emphasizes the absolutistic terms of the Eleatic concept of being, though she construes the 'is' as predicational rather than as existential). In this manner antigradualism made its first appearance.

8c.- Anaxagoras and the Principle of Universal Mixture

Now that we have attended the inauguration of one of the greatest currents in the history of philosophy, let us return to its antagonist. There is one formulation of our transition problem whose author most likely is Anaxagoras. In effect, fragment 10 asks:

> How can hair come from what is not hair, or flesh from what is not flesh? (Burnet: 259).

The difficulty of the question can be highlighted if we remind ourselves of one Parmenidean presupposition operative here, namely, that it is impossible for being to come to be from non being. Anaxagoras' answer is partly based on the observed fact that when we eat vegetables or bread, the food stuff is transformed into our flesh and bones. It seems that this observation prompted his most important claim that, as it happened at the beginning of the world, so too now, all things are necessarily together, not completely separated (Schofield: 93, 110-12); and this blending is so profound that in each thing there is a portion of everything (Guthrie 1969: 276, 286-88; Barnes 1979/2: 28-32). This last thesis has been called the '*principle of universal mixture*'(PUM). Let me intercalate one quotation. Fragment 6 declares that:

> ...all things will be in everything; nor can they exist separately, but all share a portion of everything. ...none of them could be separated, nor come to be on its own; but as in the beginning so too now all things must be together (Schofield: 105).

But the problem is that PUM cannot be taken in its full generality for it seems to command the admission that, if everything is everything, then the person with zero hairs on her scalp would also be hairy. So, it appears that, if we want to avoid the trivialization of the theory, we need to exclude the entities superlatively exemplifying a property from the scope of application of PUM. What is important and needs to be insisted on is Anaxagoras' response to the problem of change, which is that the effect or the result of the change, i.e., what is to be explained, preexisted already in the origin, but in an imperceptible manner (Cfr. Barnes 1979/2: 38), due to the mixture of everything, or because of its very small size, as Fragments 1 and 4 (second half) affirm. Furthermore, the opposites are not an exception to this general law: they too are mixed. Thus, in Fragment 8 we read:

> The things in the one world-order have not been separated apart from each other, not yet chopped apart with an axe, neither the hot from the cold nor the cold from the hot (Schofield: 105).

And this is confirmed by Aristotle's (1999: 187a, 32-33) testimony:

> ...since the opposites come from each other, they must have been present in each other (Kirk, Raven, and Schofield: 370).

From Anaxagoras we can retain the following ideas: (1) that the entities of the world, opposites included, are not separated, but, on the contrary, are somehow intimately united; (2) that, consequently, they share their own being with all others –with the restriction mentioned above; (3) that the end product of the change is at the onset of the change, but in a small degree. Thus, Anaxagoras too presents us a vision of reality in which all things, and not only the opposites, are mingled or blended.

8d.- Plato and Degrees of Being, Non-Being, and the Intermediate Things

Continuing in the same contradictorial gradualist tradition, we have to study Plato. First, there is textual evidence that Plato spoke literally of degrees of existence. He held that things are more existent the more self-identical and unchanging they are. For example, in the *Republic* IX (585b-e), he affirms that an intellectual entity may

> have a greater share of pure existence

whereas a sensual one

> participates in less real being.

Something

> «has more existence»,
> «has a more pure being»,
> «has a more real existence»

the less invariable it is (Plato 1892: 297-8). So, a particular entity may participate in the Form of Being in degrees in exactly the same way as it may gradually participate in any other Form. Existence is gradual (Runciman: 21-3, 66; Cross and Woozley: 145, 160, 175, 177-8, 184-5; Guthrie 1975: 495-97; pace Vlastos [1965], [1966]. For a criticism of Vlastos see Code).

Second, there are correlative degrees of non being. The multiple inexistent objects are not reduced to a pure nothing; what does not exist is not the same as what does not exist at all. Plato knows very well the crucial difference between two kinds of negation: one thing is to deny weakly, and another is to deny something totally. This is evident at the end of the *Republic* V (475e-480a), concerning the object of opinion, where the simple οὐκ and μή are distinguished from the strong μηδαμῇ, and παντως μὴ.

Still more clearly the distinction between two negations of being plays a protagonist role in the *Parmenides* contrasting hypotheses V and VI. From the same hypothesis «if a One (one thing) does not exist» different consequences follow according to whether the negation is taken in a soft or in its fullest possible sense. Thus, in the former case (160 c-e), from «a One does not exist», it follows that it is knowable, has a different character, and must have being (Plato 1996: 95). But in the latter case (163c - 164b), we have that:

> The words 'is not' mean simply the absence of being from anything that we say is not. ... The words mean without any qualification that the thing which is not in no sense or manner is, and does not possess being in any way (163c, Cornford's translation, pp. 231-32).

So, an absolute non being cannot be knowable, nor can it be the subject of a discourse, and it cannot have a name, nor any character whatsoever. And the same duality of non being reappears in the *Sophist*. Beside the Eleatic non being (237b), identified with nothing, Plato wants to posit another sort of non being, but only partial, one that does not preclude the inexistent entity from having some properties (Owen: 113, 118, 122, albeit this author reads the 'is' as copulative rather than as existential). That this partial non being is identical with the Form of Difference -as the majority of interpreters have held- is a disputable question, but we cannot enter in the debate here.

Third, between the "two" extremes of pure being and absolute non being, there lies a large set of intermediary objects, the beautiful things that are ugly, the just things that are also unjust, etc., all of which are and are not, partaking of the characteristics of both extremes. The objects of opinion in the *Republic* «occupy a midway position on a scale between being and not being...» (Seligman: 19, though this author denies degrees of reality in the *Sophist*). Plato's ontology is gradational, recognizing «a third intermediate region of things that are neither wholly real nor utterly non-existent» (Cornford, in Bluck: 66). So, what is intermediate is contradictory.

Let me finish with a passage from the *Republic V* (477a, 479a-b, d, Jowett's translation):

> ...if there be anything which is of such a nature as to be and not to be, that will have a place intermediate between pure being and the absolute negation of being?
>
> Yes, between them. ...
>
> Will you... tell us whether, of all these beautiful things, there is one which will not be found ugly; or of the just, which will not be found unjust; or of the holy, which will not also be unholy?
>
> No, he replied; the beautiful will in some point of view be found ugly; and the same is true of the rest. ...
>
> Thus, then we seem to have discovered that the many ideas which the multitude entertain about the beautiful and about all other things are tossing about in some region which is half-way between pure being and pure not-being?
>
> We have.

8e.- Gradualist Aristotle?

Finally, let us take a look at Aristotle [1952], who traditionally is regarded as supporting the line inaugurated by Parmenides, as long as they both deny degrees of being, and hence defend a maximalist ontology. However, there are some passages in the Aristotelian corpus expressing gradualist ideas, on which we will concentrate (Cfr. Morrison). Thus, concerning a thing that is changing from white to non white, he says:

> the fact that it is not *wholly* in either condition will not preclude us from calling it white or not-white. We call a thing white or not white not necessarily because it is wholly either one or the other, but because most of its parts or the most essential parts of it are so: not being in a certain condition is different from not being wholly in that condition
> (*Phys*. VI, 9: 240a, 21-26).

Here Aristotle advocates that a thing can be *F* even if it is not completely *F*, hence he opposes the maximalist demand. Notice also his explicit distinction among two sorts of non being *F*, where the difference appears to be gradual. Similarly, he unequivocally defends a version of what we will call the *Acquiescence, or Acceptance Rule*:

> ...any predicate of which we can speak of greater or less degrees belongs also absolutely... (*Top*. II, 10: 115b3).

In other words, what is more or less *F* is *F* (without qualification). He explains why this is so: a quality *F* will not be attributed to some extent to an object that is not *F*. So, if we put the two foregoing passages together, what the Stagirite is asserting is that in order for a thing to be *F*, it is not necessary that it be totally *F*, since to be so in a considerable degree suffices.

Next consider what the status of the intermediates between contraries is.

> There are differences of degree in hot and cold. ...when neither exists in the full completeness of its being, but both by combining destroy one another's excesses so that there exist instead a hot which (for a 'hot') is cold and a cold which (for a 'cold') is hot; then what results from these two contraries will be... ...an 'intermediate': and this 'intermediate', according as it is potentially more hot than cold or *vice versa*, will possess a power-of-heating that is double or triple its power-of-cooling...
> *(Gen. et Cor.* II, 7: 334b8-16).

The text begins by establishing the existence of degrees whenever we depart from the extremes and enter into the combination of both. The case at hand is one in which the intermediate possesses one opposite more than the other. Now, if we apply here the rule for comparatives quoted above from the *Topics* (II, 10), it follows that the intermediate has both opposites, and this explains its having both a power of heating and of cooling, but not in the same intensity. The importance of this third quotation lies in that it makes a connection between degrees and overlap of opposites. Corroboration of the mingling theory of intermediates comes from the following two cites:

> ...things exhibit such and such a character in a greater degree if more free from admixture with their contraries; e.g. that is whiter which is more free from admixture with black (*Top*. III, 5: 119a27-28).

> ...a thing's possessing a quality in a greater or in a lesser degree means the presence or absence in it of more or less of the opposite quality (*Phys.* V, 2: 226b8-9).

A *law of inverse covariance of opposites* (ICO) is stated here:

> the more an object is *F*, the less non *F* it is.

The hotter x is, the less cold it is. But again, by the rule for comparatives, x is both hot and cold. Therefore, the intermediates are somehow, or somewhat, both contraries. (*Phys.* V, 1: 224b31-34; *Met.* X, 7: 1057b24-27). Again we discover here that there where there are degrees, we are bound to find a mixture of contrary properties.

Within this context, nothing is more natural than to expect a gradual theory of change. In fact,

> that which is losing a quality has something of that which is being lost, and of that which is coming to be, something must already be

(*Met.* IV, 5*:* 1010a18-19; Aristotle 1971: 109, Kirwan's note at 1010a15; Irwin: 551, n. 27).

Does all this mean that Aristotle accepted the actual compresence of opposites in the same subject? Of course not. His main thrust is the antipode of Heraclitus, Anaxagoras and Plato. His championing defence of the principle of non contradiction forces him to do away with any road leading to actual contradictions. Most probably then, Aristotle renounces the blending conception of intermediates: these are a *tertium quid* with respect to the extremes; the intermediate is the negation of both contraries (*Cat.* 10: 12a19-24; *Top.* IV, 3: 123b23; *Gen. et Cor.* II, 7: 334b27; *Soph. Ref.* 5: 167a16-20). Thus, Aristotle ends up rejecting the coalescence of opposites: they do not mix. Indeed, in the final analysis, Aristotle gets rid of degrees altogether, since, though it would be blindness not to acknowledge them, their full admittance would land us in contradictions. Thus, substances do not admit of degrees within themselves (*Cat.* V: 3b32-4a9). Therefore, he replaces the gradational appearance and disappearance of entities by the dichotomy of act and potency, the plurivocity of being (*Gen. et Cor.* I, 3: 317b16-18).

A summary is presented in the following two columns, each line representing two contrasting modes of conceiving being, the relations among the opposites, and the transition from one to the other. The left column shows how reality is like according to the friend of gradual contradictions, while the right column depicts the dichotomist framework. These are the main options: contradictorial gradualism vs. discontinuism.

	FUZZY REALITY	DISCONTINUIST THINKING
BEING	degrees	modes or kinds
	univocism	plurivocism
	more or less	all or nothing
	intermediaries	extremes only
OPPOSITES	overlap	total exclusion
	contradiction	sharp division
	both	either... or
	conciliation	absolute incompatibility
	coexistence	separation
	compresence	disjointness
	harmony	clash
	homogeneity	dichotomy
	union	duality
TRANSITION	gradual	discontinuous
	smooth	abrupt
	bridge	abyss
	continuity	cut

9.- Summary

When one approaches the problems of fuzziness and the sorites from the perspective of a many-valued and paraconsistent logic, one is in a privileged position, for, unlike in classical logic, one is able to make distinctions, and therefore, qualified assertions. Indeed, a language based on degrees of truth and the toleration of contradictions has a richer expressibility power. In sharp contrast, classical logic is severely limited in that it cannot properly convey, for example, the relation among members in a soritical series, since not only it cannot distinguish between the Parity and the Preservation principles, but also it cannot formulate a principle that is true of the soritical series.

The main ideas put forward in this introductory chapter have been the following. Fuzziness is acknowledged as being softly, but not entirely, indeterminate. As a result, the simple Principle of Excluded Middle is partially false, though it remains also true, to some degree. But the absolute PEM has to be given up. It has been urged that fuzziness is contradictory, has many mild limits, and generates the sorites paradox, but it is not to blame for the absurd conclusion.

Concerning the sorites, we have expounded two grounds for the plausibility of the major premise: the *G-F* Correspondence Principle, and the Fairness Principle. On the other hand, the rejection of the major premise (discontinuism) and the abandonment of the rule of inference of the slippery slope (maximalism) are fraught with serious problems. What goes wrong with the sorites for a bounded series is the rule of inference: disjunctive syllogism is not valid for the weak negation. And in the case of an unbounded series, we need to accept the conclusion: everything is *F*.

We will have occasion in the following chapters to return to these topics. In the next four chapters, 2-5, we will expose the main alternative theories of vagueness. Chapter 6 will be an attempt to argue against those views that reject the major premise of the sorites and in favour of contradictorial gradualism.

CHAPTER 2

AGNOSTICISM

In this second chapter, we begin examining various responses to our two topics in contemporary analytical philosophy. We start with a position that has been called epistemicism, whose main advocates are Timothy Williamson and Roy Sorensen. This chapter also includes the views of the late Willard van Orman Quine, and discusses a critique of bivalence by Michael Dummett.

1.- Quine

Quine has characterized vagueness in his masterpiece *Word and Object* [1960]. The vagueness of a term has to do with its possessing «fuzzy edges» (125). The term 'green' is vague to the extent that it is unsettled how far a green thing can go toward the blue zone of the colour spectrum or toward the yellow one and still be counted as green. The vagueness of words stems from the mechanism of their learning process. The set of stimuli prompting a verbal response that is rewarded is not a clearly delimited class but a distribution around a norm. The closer to this norm a stimulus is, the stronger the disposition to elicit the appropriate verbal response will be (85). The penumbra of a vague term is thus constituted by those objects bearing a relatively low similarity with those objects in front of which a verbal response was reinforced.

Quine says that vague expressions usually do not perturb the truth value of sentences in which they appear, though they are the source of concern in specialized fields (128).

On the other hand, Quine's objective in his article "What Price Bivalence?" [1981] is to acknowledge the costs of his allegiance to the Principle of Bivalence. He explicitly employs a strong version of the PB, which demands that every sentence «be univocally true or false» (91). Circumstances must be such that they speak with one voice at the moment of deciding which truth value a sentence has. In other words, a general word «must be definitely true» or definitely false of every object (94). The main reason for embracing the PB and the whole of classical logic -Quine says- is the simplicity they provide. Yet, avowedly, their cost is not low.

The problem with vague terms is that they generate the sorites paradox, and therefore beget contradictions (91-92). In order to avert the absurd conclusion, Quine is compelled to renounce one of the major premises of the argument. As loyal partisan of CL, he is ready to advocate the Discontinuity Thesis, $\exists a_i, a_{i+1}$ $(\Delta Fa_i \wedge \neg Fa_{i+1})$, accepting that fuzzy terms have sharp boundaries. Let us imagine a soritical series representing the process of destruction of a table, such that each subsequent member of the series has one less atom than its predecessor. We ask ourselves, at what moment the table is not a table any more. The PB forces a bipartition of the series.

> If the term 'table' is to be reconciled with bivalence, we must posit an exact demarcation, exact to the last molecule, even though we cannot specify it. We must hold that there are physical objects, coincident except for one molecule, such that one is a table and the other is not (94).

The PB exacts that vague expressions be made precise. We must fix the meaning of terms like 'bald' and 'heap' by stipulations that are arbitrary, as Quine admits. That is, we must specify how many grains of sand are necessary to constitute a heap, and what the maximum number of hairs a person has to have to be bald, and so on. (It is presupposed that the rest of the terms that serve to define the original word have themselves precise borders. Quine concedes that this is a fiction). In this sense, the determination of the meaning of vague terms is not a matter of fact, settled by the disposition of physical states, but a matter of convention. Notice that precision is an imperative even in the case where we cannot find a way to carry it out, when the situation is non specifiable. This is what happens with 'table': there is no way to delimit its meaning, not even by an arbitrary stipulation.

One of the results accepted by Quine of this precisification technique is that what were observational terms before become now theoretical, since their application «in marginal cases» (92) will depend on tests and inferences, rather than on sheer experience alone. That is, the application of the vague term to dubious cases will depend on our verifying that its condition of application is met. For example, supposing that 'bald' is stipulated to mean having less than 10,000 hairs on one's scalp, I would have to count how many hairs Frank has if I think his number of hairs lies in the vicinity of that threshold.

However, from our point of view, the highest cost of respecting bivalence is that we are compelled to attribute a general term to, or withhold it from an object «even in the absence of ... [an] objective fact» (94). I.e., the principle of bivalence has the consequence of positing truths without their corresponding facts in the world. The truth maker principle is not valid.

1a.- Evaluation

Quine's position has merit in as long as he has made it clear what the consequences of his championing defence of the strict Principle of Bivalence are. We have seen that, *if* the strict PB is tenable, *then* fuzzy words have to be rendered precise, resulting in the surrender of the major premise of the sorites, and the support of the Discontinuity Thesis. We think that this conditional is indeed correct. However, reasoning by *modus tollens*, we question the truth of its antecedent. That is, given that the strict PB entails the abandonment of the major premise of the sorites and the Discontinuity Thesis, we should discard the strict PB. But bear in mind that we still support a weaker version of the PB.

On the other hand, the precisification project saddles us with unbearable burdens, such as its arbitrariness and the loss of observational terms. The first mentioned inconvenience means that we lack a reason to prefer a_i over a_{i+1} or a_{i-1}, as being the last element to be F, violating the ***Principle of Equity***:

> to treat in the same manner equal cases, and in an approximate manner approximately equal cases.

There is no fairness when the decision turns on the whims of the judge. And the precisification of the fuzzy adjective 'bald' would entail that in the unclear cases we have to count the number of hairs of a person before we are able to apply it. But this would destroy the whole point of fuzzy terms, which do not require any such measurement. Rather their very intent is to bypass it.

2.- Dummett vs. Strong Bivalence

Dummett [1995] launches an attack against the strong version of the Principle of Bivalence from an indeterminist stand-point.

The concept of a vague expression employed by Dummett is the standard one: an expression 'F' is *vague* if it has the possibility of borderline cases, i.e., cases which are neither definitely F, nor definitely not-F. Dummett offers a second characterization of borderline cases, making reference to the speakers of the language who may assert either that the object x is F or that x is not-F, without displaying any lack of competence in the use of F.

Each vague expression is associated with three sets not well delimited among themselves: (1) a (positive) extension, comprising all objects which are definitely F, (2) a negative extension or antiextension, encompassing all elements which are definitely not F, and

(3) the penumbra, containing the borderline cases. On the other hand, an expression is *precise* if it is not vague.

Let me now explain what the *precisification* involves. To precisify a fuzzy word '*F*' is to allocate its borderline cases into either its positive or its negative extension. The following chart illustrates the procedure.

fuzzy word: Δp ... not $\Delta p \wedge$ not Δ not p ... Δ not p

precisification: borderline cases

precise replacement: Δp | Δ not p

If, as a result of the relocation, no penumbral cases of the original word remain at the end, then the precisification is *total*, and the new term is precise; otherwise, if the operation has managed to just reduce the number of intermediate cases while still leaving some in the penumbra, then the precisification is *partial*, and the new term continues being vague.

Then Dummett defines *precisionism* as the position which holds that all vagueness can be eliminated, that is, it maintains that every vague expression lends itself to a total precisification. All borderline cases of *F* can be assigned either to its extension or to its antiextension, without residue. So, precisionism effects a reduction of fuzzy terms to precise ones, and thus it intends to save the two-valued semantics. What is important to note is the conception of reality behind this stand. For precisionism, reality is wholly determinate, or determinately constituted. Dummett explains the precisionist vision of the world in heuristic or epistemic terms: every well-defined question about the world has a determinate answer, known or unknown to humans. It is clear that the defender of the principle of strong bivalence is a precisionist.

On the other hand, *imprecisionism* defends that there are vague expressions which cannot be totally precisified. One example Dummett gives is the predicate 'amusing', of which it appears preposterous at first glance to maintain that it can be completely precisifiable (208). In other terms, reality cannot be fully described without using vague vocabulary. Something would be lost if all we have to say were precise.

What exactly does the Principle of Bivalence state? According to Dummett, part of what it says is that:

> Every unambiguous expression is capable of total precisification; and every unambiguous, precise statement is determinately either true or false (211).

By narrowing the scope of application of the PB to precise sentences, vague ones are explicitly excluded. The reason for this exclusion is in line with the precisionist's conviction that vagueness is not a feature of reality but only a defect of language.

Dummett disagrees. If there is a term which is not totally precisifiable, then its vagueness is not merely a matter of language but of reality (208). But if there is vagueness in reality, it means that reality is indeterminate, or that the indeterminacy is real (215). Yet, Dummett denies that there can be vague objects, though there can be objects with vague boundaries. For instance, the question: 'Are we already in the Gobi desert?' may not have a determinate answer; and this merely entails that the predicate 'is in/on a part of X' is vague. Thus, properly speaking, vagueness is not a property of objects, but of expressions for properties and relations, Dummett argues (210).

He also contends that the fact of precisifying vague words will destroy their whole point, as it has been the case with the concept of 'intelligence', whose point has been obliterated by psychologists.

If the debate is framed as the alternative between precisionism vs. imprecisionism, then I side with imprecisionists. However, if the parties to the dispute are strict bivalence and indeterminism, then I think the dilemma can be avoided taking a third line: fuzzy sentences do not have a classical value, and on this particular point indeterminists are correct; but there is a large supply of degrees of truth, which allow us to make assertions less than absolutely and determinately true, and therefore, more nuanced and qualified. Yet, a weak version of the principle of bivalence is acceptable provided that we drop the thesis of existence of just two truth values. Leaving this contentious presupposition out, we can perfectly maintain that every sentence is either true or false. Thus, we scape both extremes, taking a middle course.

Let me end this section by quoting the concluding remarks of Dummett concerning precisionism and strong bivalence:

> the creed appears a wild metaphysical extravagance; but it is what we have all been brought up to think, and has taken such deep root that many philosophers with little imagination dismiss as fatuous any attempt to question it (216).

3.- Williamson's Agnosticism

Timothy Williamson is without doubt the most famous author who has written extensively on our two topics, and in a highly rigorous way. Several philosophers have taken issue with his agnosticist theory, to some of whom Williamson has replied. We will review the more salient ideas of this lively debate. But first, let us give a summary of the main components of his approach.

3a.- Basic Tenets

Part of Williamson's motivation for developing agnosticism is his desire of maintaining the whole of Classical Logic and semantics. The first sentence of the Preface of his [1994b] *Vagueness* is this:

> This book originated in my attempts to refute its main thesis: that vagueness consists in our ignorance of the sharp boundaries of our concepts, and therefore requires no revision of standard logic (p. xi)[8].

And on the Introduction, he says: «The thesis of this book is that vagueness is an epistemic phenomenon. As such, it constitutes no objection to classical logic or semantics» (p. 3). Hence, according to him, vagueness does not provide any cogent reason to revise CL. The Principle of Bivalence applies to vagueness: each vague statement is either true of false. Furthermore, one of the grounds advanced in favour of bivalence is that CL is superior to its alternatives in power, simplicity, past success and integration with theories in other domains. Williamson thinks that on these grounds alone, it is not irrational to claim that bivalence should also apply to vagueness (186). So, it is his desire to defend CL and its PB that has prompted the development of agnosticism.

Williamson has made two cases in support of the principle of bivalence. One is to derive PB from the principle of excluded middle, plus the disquotational principles for truth and falsity, and the definition of the falsity of a sentence as the truth of its negation (1992: 145). In fact, from $p \vee \sim p$, and $Tp \equiv p$, $T\sim p \equiv \sim p$, we get the subconclusion that $Tp \vee T\sim p$; and

---

8 Unless otherwise noticed, references throughout this section 3 are to his classic [1994b].

from here, $Tp \vee Fp$ is finally deduced (1994a: 174). A second case is presented in Andjelković and Williamson (2000: 211) for a strong version of *bivalence*, to wit:

> truth and falsity are jointly exhaustive and mutually exclusive.

The general claim made there is that the thesis that, relative to a context c, a sentence is not true if and only if it is false is inferred from principles characterizing truth, falsity and negation (221).

Vagueness is essentially an epistemic phenomenon, consisting in our ignorance of which classical truth value a vague utterance bears. These are the defining features of *epistemicism* (202; 1997b: 921). When Timothy is a borderline case of thinness, the utterance 'Timothy is thin' is either true or false, but there is no way for us to discover which of these holds. Our inability to find out is not merely a matter of fact but one of principle; the kind of ignorance involved in vagueness is just a special case of the more general phenomenon of inexact knowledge, as we will see later in the next section. Accordingly, a borderline case satisfies the following disjunction: either 'p' is unknowably true, or 'not p' is unknowably true (1994a: 175).

To be more specific, the ignorance involved in vagueness is at least fourfold, for we do not know: the sharp boundaries of the expression or the concept '*F*' (234), nor of its ontological correlate; for instance, we cannot know how many grains make a heap (1997b: 926); and therefore, we do not know whether the borderline case *a* is *F* (185), nor the truth value of the vague statement '*Fa*' (201).

The procedures which are normally used to decide clear cases of a predicate do not help us in borderline cases. So, '*vagueness*' can be defined as unclarity (p. 2): it is not clear whether Timothy is thin or not. More accurately, it is unclear whether Timothy is thin, and it also is unclear whether Timothy is not thin. We do not have a clear idea of what state of affairs is actual. We can define a borderline case by means of the definitely operator, 'Δ', as long as we give it an epistemic interpretation: x is neither definitely *F* nor definitely not-*F*; that is, x is neither clearly *F* nor clearly not-*F*. More rigorously, 'Δp' may be explained as the absence of obstacles of a special kind to knowing that p (2004: 118).

Williamson alleges that the precedent definition is neutral with respect to the question of the origin of the unclarity, i.e., the definition does not make a pronouncement as to whether the lack of clarity is due to an objective indeterminacy or to our subjective deficiencies.

By the way, it seems that the denomination of 'epistemicism' is a kind of euphemism, or misnomer, since vagueness is characterized in terms of ignorance. In order to explain the ignorance, a particular conception of knowledge enters the scene; however, this is not a definitional ingredient of vagueness. Since the impossibility of knowledge is indispensable, we judge 'agnosticism' is a name better suited for the position.

A second characterization of vagueness is that an expression '*F*' is *vague* when there are possible expressions which speakers cannot semantically discriminate from *F*. The greater the indiscriminable difference, the greater the vagueness (237). For example, the predicate 'thin' could very easily have another extension slightly different from the actual one. Thus, what is distinctive of vague words is that their meaning is unstable (217). Vague concepts have sharp but unstable boundaries (231).

Notice that agnosticism is a sort of subjectivist position, insomuch as the nature of vagueness necessarily involves a reference to our cognitive limitations, and thus in a sense it is mind dependent. The source of vagueness is our limited capacity of discrimination:

> Vagueness issues from our limited powers of conceptual discrimination (237).

As Williamson (2003: 712) acknowledges:

> Epistemicism provides only an aetiolated sort of metaphysical vagueness, constitutively dependent on thinker's epistemological limitations.

However, there is a sense in which agnosticism permits objects to be vague, in as long as «the impossibility of knowing their boundaries may be independent of the way in which the objects are represented» (6). I take this passage to mean that our ignorance of how to classify a borderline case x does not turn on how we call it. On the other hand, reality itself is precise, with all entities having sharp boundaries (*Ibid*. 690, 710).

According to Williamson, one central feature of vagueness is second order, or higher order vagueness. *First order vagueness* refers to the unclarity as to whether the state of affairs p or its negation, not-p, holds. *Second order vagueness* refers to the classification of the states of affairs of the first order classification as either definitely holding, definitely not holding or borderline (1999a: 132). That there is second order vagueness means that the frontier between the unclear cases and the clear cases is unclear. Not only it is indefinite when Rembrandt has become old, but it is also indefinite when he has become definitely old. One aspect of second order vagueness is that the scope of borderline cases is itself not clearly delimited, so that it is vague which cases are vague; there are borderline cases of borderline cases.

As for the sorites, the main theses defended by Williamson are the following. The argument is valid, but not sound. If we want to prevent the paradox, we must reject a premise. In view of the falsity of the conclusion, one of the conditional premises must be false, though we are unable to pick out exactly which one is faulty. This means that for some unknowable a_i in the soritical series, a_i is *F*, but a_{i-1} is not *F*. There must be a point at which the removal of one single grain turns the heap into a non heap; otherwise, the removal of all grains will leave us still with a heap! So, a_i is the last to be *F*, and a_{i-1} is the first to be not-*F*. One grain does make the difference between what is a heap and what is not a heap. There is a sharp cut-off point in the series, whose precise location we cannot pinpoint. Williamson mentions that this bipartition of the soritical series into a positive and a negative sections follows from the principle of bivalence (p. 1). At any step, either it is true that the object a_i is *F*, or it is false, and therefore, the object is not *F*.

3b.- The Unknowability of the Sharp Boundary

In this section we will see Williamson's explanation of why it is that we cannot know the sharp boundaries of vague expressions. For him the ignorance engendered by vagueness is simply a special case of inexact knowledge. Williamson asks us to compare two situations, *i* and *j*, such that the former is *F*, and the latter, not-*F*, and, in those circumstances, to consider what the conditions for knowledge are. For instance, suppose that the exact number of books on a shelf is *i*. By looking at it, I can come to truly believe that there are not *j* books there. Now, Williamson claims that the true belief that there are not *j* books constitutes knowledge or not depending on what the difference between *i* and *j* is. If that difference is large enough, then my believe is knowledge; but if the difference is too small, then my belief is not knowledge, because it would be unreliable. More concretely, assume that there are exactly 1499 books on the shelf. If you believe that there are not 3000 books, you have knowledge; and again if you believe that there are not 2500. But if you truly believe that there are not 1500 books, your true belief could very easily turn into a falsehood if by chance there was one more book on the shelf, in which case your belief could not qualify as knowledge. Knowledge is not a matter of sheer luck or happy accidents. So, if the difference between a situation p and another not-p is so narrow that I cannot discriminate them, there is no knowledge; there is knowledge only if the difference between a situation in which the content of the knowledge obtains and other situation in which it does not obtain is wide enough.

Williamson draws the consequence that, in order for a belief to be knowledge, it has to possess a margin for error (MfE); that is, the variation allowed between the situations *i*, which are *F*, and those *j*, which are not *F*, must be larger than a certain range, at least much bigger than our threshold of discrimination; whereas, if the variation between *i* and *j* is smaller than that range, then the belief fails to be knowledge. If a true belief is knowledge, then it cannot become false by a tiny, unnoticeable change in the objective circumstances. Knowledge does not have such kind of instability. In other words, what one knows ought to remain being the case even when the situation prompting the belief has been altered a little, for otherwise, knowledge would not be reliable. That is, knowledge that x is *F* is available if its content obtains in all cases x' similar to x. What is necessary for the belief that x is *F* to exhibit a margin for error is that all x' similar to x be also *F*. Therefore, inexact knowledge need to have a MfE. Notice that this conception of inexact knowledge is grounded independently of which position one adopts regarding the problem of vagueness.

Basically from this conception it follows that one cannot know whether an object x is *F* when it is located very close to the borderline, for, in that case, it might be indistinguishable from a situation in which it is not-*F*, as we will immediately demonstrate.

Williamson formulates a *Margin for Error Principle* (MfEP, from now on) for the case of vagueness, namely, if one knows that an object *i* is *F*, then all objects *k* similar to *i* are also *F*.

Yet, there is no single, uniform, or canonical form of the principle, the difference lying in the presence or absence of the word 'truth' or 'true' in it. To see this minor variation, let me quote some versions of the MfEP. The first published formulation is this:

> If *x* and *y* differ in physical measurements by less than *c* and *x* is known to be thin, *y* is thin (1992: 161),

where *c* is a small but non-zero constant. The official phrasing of the MfEP appears in *Vagueness* (227):

> 'A' is true in all cases similar to cases in which 'It is known that A' is true.

Though here the word 'true' occurs twice, it seems to me that it is redundant, as it is shown few pages later (232):

> '*n* grains make a heap' expresses knowledge only if '*n*-1 grains make a heap' expresses a truth. In other words, a margin for error principle holds:
> (!) If we know that *n* grains make a heap, then *n*-1 grains make a heap.

Thus, though Williamson does not, we could symbolize the principle as:

$$\text{(MfEP)} \quad Kp \supset Tp'$$

where 'K' stands for 'it is known that', "p'" is a sentence referring to a situation similar to that meant by "p", and 'T' is the truth predicate, which is not essential, and may be dispensed with, as we have just seen.

It is this principle (MfEP) that discharges the duty of justifying the unknowability of the crisp border of vague concepts. In fact, to know the sharp border $Fa_i \wedge \neg Fa_{i-1}$, the left conjunct would have to be known; that is, 'KFa_i' would be true. If that were so, by the MfEP, 'Fa_{i-1}' would be true, falsifying the right conjunct, and hence the whole conjunction too, its falsity preventing it to be known (233; 1996b: 40).

Our ignorance of the sharp boundary of a vague concept is not merely a matter of fact, but a matter of principle. Yet knowledge of the exact location of the cut-off point is not a metaphysical impossibility (198-201; 1997b: 926; 1996b: 41). As it was expected, we humans do not have knowledge when the difference between a thing which is *F* narrowly differs from another which is not-*F*. The link between vagueness and ignorance is so essential that, if we can know of a concept the turning point marking the division between the objects falling within its extension and those falling outside, then the concept has no borderline cases, and therefore it is not vague (2002a: 149).

On the other hand, nobody can know that x has a vague property *F*, if x is a borderline case of *F*. If Frank is a borderline case of the property bald, then that Frank is bald is unknowable. Where '*P*' is a vague sentence about a paradigmatic borderline case, «assertions and denials that *P* are not expressions of knowledge» (1994a: 174).

3c.- The Supervenience of Vagueness

Williamson distinguishes two respects in the base on which vagueness supervenes. First, vagueness supervenes on exact facts; that is to say, the former is determined by the latter. We will use the symbol '*G*' to refer to the supervening base. The notion of *supervenience* is roughly explained thus. If the property thinness supervenes on the precise measures of the body, then there cannot be two persons with exact physical measures such that only one is thin while the other is not. In general, there is no change in the supervening vague property without change in the subvening precise circumstances. Hence, on equal subvening exact situations, there supervene equal vague properties. And by contraposition, there is a difference with respect to a vague predicate only if the supervening base is different. What is peculiar to agnosticism is that no amount of information about *G* will enable us to know the status of the property that supervenes on it. That is, no matter how well I am acquainted with the measures of Timothy's body, that I will not learn whether Timothy is thin or not. There is no way to get knowledge of *F* from knowledge of *G*.

Remark also that this notion of supervenience does allow the possibility that two objects differ minimally in the underlying dimension *G* and however be such that only one is *F* whereas the other is not *F*. By way of example, there might be two persons differing in the measure of their waists minimally, say by at most 1 mm., and such that only one of them is thin but not the other.

And the second aspect of the supervenience relation is that (vague) meaning supervenes also on use, plus environment. Leaving the contribution of the environment constant, what is essential is that there be no change in meaning without a change in use; equal use entails equal meaning; and different meaning entails different use (206; 1996c: 330-31). However, the supervenience relation between meaning and use is a little more complicated. By his conception of inexact knowledge, Williamson is obliged to admit that a slight change in *all* our dispositions to use a word would slightly change its meaning (231; 1992: 160). Nonetheless, he is also forced to restrict the general validity of this supervenience relation since it is not always the case that a minimal difference in the use of a term by two speakers implies a minimal difference in meaning, because meaning is socially determined (236; 1994a: 186).

> Perhaps no two speakers of English match exactly in their dispositions to use 'thin'. It does not follow that no two speakers of English mean exactly the same by 'thin'. For what individual speakers mean by a word can be parasitic on its meaning in a public language. The dispositions of all practitioners collectively determine a sense that is available to each (211).

Williamson answers in advance to an objection to his theory. According to the objector, agnosticism has to drive a wedge between meaning and use, since the meaning of vague words would have sharp limits, whereas the practice of users does not establish any such precise borders. In general, any theory which postulates an exact boundary for the vague expression '*F*' will divorce itself from the use of such expression, since competent subjects do not draw crisp borderlines to delimit the meaning of vague words. Williamson replies that there is no simple-minded reduction of meaning to use, and that the supervenience relation is such a chaos that it is unsurveyable (209). For one thing, a complete survey of all the data is unmanageably complex for an individual and worse for the society as a whole. For another, even if the survey could be carried out, there is no systematic procedure to infer semantic facts from the data about dispositions to use words. «If use determines meaning, it does so non-algorithmically» (1994a: 184-85).

3ci.- The Discussion on the Determination of Meaning

We now explore the discussion around the determination of vague meaning in Williamson's theory.

OBJECTION 1, by Wright [1994].- For Williamson, the referent of vague predicates are exact properties. To be tall is to measure at least -say- 1.8 m. But do we know specifically which exact property 'tall' designates? Despite Williamson's denial (1994a: 183) that this is a fruitful question, Wright objects that Williamson has not offered any account of what constitutes the reference of our vague terms. How is it that our dispositions, intentions, or use, or its causes, or whatever, fix that the minimal threshold of 'tall' is set at 1.8 m? Agnosticism makes us ignorant not only of the threshold but also of the nature of the facts that determine the precise reference of vague expressions. And, for Wright, this is not to have a theory of reference. For Williamson, then, although we know that the adjective 'tall' denotes the property tallness, which precise property this is remains beyond our ken. But some evidence is required in order to substantiate the claim that it is impossible to know which exact entity is the referent of vague expressions.

ANSWER by Williamson [1996b].- With respect to the accusation of an absence of a theory of reference, Williamson responds that his agnosticism is compatible with almost any theory of reference, except verificationism, which conflicts with his realism.

For Williamson's response to an aspect of the objection dealing with the demand for a demonstration of the impossibility of our knowledge of which exact entity corresponds to a vague expression, please see his answer to Schiffer immediately below.

OBJECTION 2, by Schiffer [1997], [1999].- Schiffer thinks that there are several problems with the agnosticist position. I enumerate some of them. He mentions that one reason for rejecting the belief that there are sharp borders is that it is incredible that, use plus environment, the base on which supervenes the meaning of vague expressions, determine such exquisitely fine-tuned referents. Consider, for example, the vague phrases in the following sentences: 'Joe worked yesterday for a little while'; 'Elizabeth stood roughly there', 'Tom added a pinch of salt to the egg'. In the first case, Williamson's theory implies that there is a precise span of time, say two hours and seven seconds, such that the statement is true if Joe worked for a period of time lasting that much, but that it is false if Joe worked one nano-second more. And similarly in the other cases. Moreover, nobody will know what the exact limits of the region of space to which 'roughly there' refers to are. Furthermore, there is no reason stemming from Williamson's views on meaning or reference to expect that there are such determining factors of the precise referents.

On the other hand, even though we did not have a systematic procedure to discover the supervenience, we still could use a method of trial and error in our attempt to find out the non intentional base on which semantic facts supervene. To simply assert that use determines meaning non-algorithmically is too cryptic a remark to constitute a satisfying explanation.

ANSWER by Williamson [1997c], [1999b].- With respect to the question of how use and environment factors determine a unique reference for the vague expression '*F*', Williamson answers that when those factors do not sufficiently determine that an object x belongs to the intension of '*F*', then they do determine that x does not belong to the intension of '*F*'. These factors do not make room for indeterminacy (1999b: 509-511). But whether they determine that x has the property *F* or not is a different matter from whether or not we know what '*F*' refers to. In fact, it is impossible to know that 'bald' designates the property of having less than 3,832 hairs on one's scalp due to the vagueness of the reference relation. The same account explaining our ignorance of the cutoff of ordinary vague expressions is applied to the case of semantic properties. The belief that 'bald' denotes the property of having less than 3,832 hairs very easily would have been put in our belief box even though the word had shifted its reference to designate the property of having less than 3,831 hairs. If so, the belief would have been turned false, and hence it could not constitute knowledge. Additionally, not because *G* is metaphysically necessary and sufficient for *F* it is guaranteed that we know that identity. Hence, we are unable to know the reference of vague terms.

On the other hand, the phrase 'precise reference' used by Schiffer is ambiguous. If 'precise' qualifies the manner of reference, then agnosticism does hold that vague words refer vaguely, not precisely, as we have just stated. But if 'precise' qualifies the object of reference, then Schiffer has not made it clear how he understands the distinction between vague and precise when it is applied to a non representational object.

OBJECTION 3, by Machina and Deutsch [2002].- According to Williamson, meaning is determined by, or supervenes on a mishmash of linguistic conventions, unstable patterns of use, changing and overlapping linguistic communities, in interaction with an evolving world. He insists that there is no algorithm to obtain knowledge of meaning from knowledge of the underlying use and environment, so that we are incurably ignorant of how the exact boundaries are fixed. And our ignorance is no evidence of the inexistence of precise borderlines. However, Machina and Deutsch object that to believe that there are precisely bounded extensions of vague predicates is to go against all natural expectations, considering the agnosticist's story of how the extensions get fixed. If meaning arises from a mishmash of intervening factors, then one would expect imprecise conditions of application for vague predicates.

ANSWER by Williamson [2002].- Williamson concedes that meaning supervenes on messy, fluctuating, heterogeneous, and casual use. Yet he does not accept that, when there is an object *a* which falls within the extension of the predicate *F*, there is some *local* factor in the supervenience base that makes *a* fall in the extension of *F*. The supervenience base may *globally* determine whether *a* falls within the extension of *F* or not by default: if the base does not sufficiently determine that *a* falls within *F*, then it sufficiently determines that *a* does not fall within *F*.

OBJECTION 4, by Mott [1998].- Williamson cannot explain how reference to a particular property is done by a certain predicate. Take, for example, the predicate 'thin'. Mott calls 'thin_0' the property of having a girth measuring no more than -say- 80 cm. The property thin_{23} will be the one whose maximum bound is 80 + 0.23 cm. There are continuum many thin properties. For each real number *r* in [-1, 1], we can have a thin_r property obtained from thin_0 by either augmenting *r* cm. to, or reducing *r* cm. from its maximum bound. For Williamson, the overall pattern of usage makes the adjective 'thin' mean the property thin_n, for some particular *n*. The predicate 'thin' must refer to a specific property out of this large collection of properties, but we cannot know which is the precise referent of the predicate. Yet the problem is that in the borderline region there is no consensus among the speakers about whether to call a person thin or not. Hence, how can we be confident that this pattern of usage picks up a unique exact property? That the predicate refers to a unique property seems extravagant.

ANSWER by Williamson.- Williamson's answer may appeal to the distinction between local and global determination, as well as to the principle of determination by default. See his response to Machina and Deutsch above for details.

OBJECTION 5, by Simons [1992], [1996].- Simons remarks that a methodological guideline should be observed. Any adequate theory of vagueness must not put meanings so far out of our grasp that we cannot explain how they come to mean what they mean. This is particularly problematic for Williamson's agnosticism, for it postulates that, keeping the contribution of the environment fixed, the overall pattern of use of a vague expression determines its meaning, but it is unable to explain how this occurs. «If ...vague facts supervene in an unsurveyably chaotic way on precise ones, then we have not even the beginnings of an account of the nature of this supervenience...» (1992: 167).

ANSWER by Williamson [1992].- Concerning the accusation of lacking a theory explaining how use determines meaning, Williamson replies that nobody has a comprehensive account of the matter, so that agnosticism is not in a worse position than others, which are equally unable to provide a detailed analysis of how reference is fixed.

If we review the debate, we discover that the most common criticism is that, given that there is a lack of consensus among speakers with respect to whether a vague expression '*F*' applies or not to a borderline case, *a*, and that Williamson himself admits that the cut-off point is not fixed by nature (1994a: 183), but instead he subscribes to the supervenience of meaning on use (plus environment), it is unbelievable that ambivalent, or hesitant pattern of usage can give rise to a precise referent for '*F*'. This is one point which has found most resistance.

Rosanna Keefe [2000], in her chapter devoted to agnosticism, arrives at a similar conclusion about the uneasiness felt with respect to the cause of the emergence of precise bounds of vague expressions. This is the greatest threat to Williamson's theory. She contends that any decision concerning the precise reference of vague words would be arbitrary (82). The task that Williamson should fulfill is not that of proving that we cannot know where the borderline is located, but rather that of accounting for why we do not believe in the existence of sharp boundaries (72). In fact, we believe that vague expressions lack such boundaries. For Keefe, this is one of the most entrenched intuitions people hold. One reason for our belief that a crisp cut-off point does not exist is that nothing could determine it. What possibly could draw the sharp line? Nothing. There is no natural bounds, and we do not stipulate where they lie (77). Keefe gathers that the supervenience thesis does not help Williamson to establish that there are sharp boundaries nor to explain how use determines them (83), nor how a unique line is chosen among alternatives. Everything rests on the demands of classical logic.

3d.- On the Alleged Incoherence of Non-Bivalence

One important argument Williamson puts forward in favour of agnosticism is that positions denying the principle of bivalence or of excluded middle are incoherent. The reasoning supposes that the bearers of truth are utterances, but not the content of what one says (a statement, or a proposition). The reason for this is that, if the bearer of truth were the propositions, then there would be agreement among defenders and objectors of the principle of bivalence. For supervaluationists hold that a fuzzy utterance 'u' expresses as many propositions as there are legitimate ways to precisify the fuzzy word. The sharpened propositions are bivalent, but not the utterances, which are neither true nor false. If agnosticists discuss with supervaluationists, that is partly because the latter maintain that bivalence fails for utterances. Hence, in order not to miss the controversy, utterances should be the truth bearers (187; 1997b: 925, n. 3).

The argument, by reductio ad absurdum, to the conclusion that to deny bivalence is incoherent, has three presuppositions, namely: the principle of bivalence, (PB), and what Williamson calls the Aristotelian conception of truth, consisting of two subtheses, (T) and (F), essential for any understanding of truth and falsehood. In what follows, 'u' is the name of an utterance, and 'p' is a declarative sentence saying that something is a borderline case. I transcribe the original proof, leaving the negation as it appears.

(PB) If u says that p, then u is true or u is false
(T) If u says that p, u is true iff p
(F) If u says that p, u is false iff not p

(0)	u says that p	Hyp., vague utterance u
(1)	not (u is true or u is false)	Hyp. for RAA
(2)	u is true iff p	(T), 0, MP
(3)	u is false iff not p	(F), 0, MP
(4)	not (p or not p)	1, 2, 3, subst.
(5)	not p and not not p	4, DM
(6)	u is true or u is false	1-5, RAA.

In line (1), u is a vague utterance which supposedly does not comply with (PB), but it does say something (line (0)). Williamson claims that (1) has been reduced to the absurdum. More precisely, it cannot be that u both says something and is neither true nor false; that is, u is indeterminate on pain of not saying anything. Line 6 shows that the borderline utterance u is bivalent on the condition that it says something.

3di.- Discussion

Before considering several lines of attack against this reductio and Williamson's replies to them, let me give my personal assessment. In this regard, I wish to challenge Williamson's claim that non-bivalent approaches are incoherent. If we evaluate the argument that the denial of bivalence is absurd in the framework of a many-valued and paraconsistent system, there are at least two readings that the argument might receive, for 'true', 'false', and 'not' may be differently interpreted. In fact, 'not' may be taken as the weak negation, and 'true' and 'false' as referring to the set of designated and antidesignated values, respectively. However, these meanings are not the intended ones. Furthermore, it is obvious that, in a classical context, 'true' and 'false' are understood as the values 1 and 0, respectively, and the negation is the strong one. So, our next task is to recast the proof so that the ambiguities are dispelled.

Classicist Interpretation

$(PB)_{CL}$ u says that p $\supset$. $/u/=1 \vee /u/=0$
$(T)_{CL}$ u says that p $\supset$. $/u/=1 \equiv p$
$(F)_{CL}$ u says that p $\supset$. $/u/=0 \equiv \neg p$

(0)	u says that p	Hyp.
(1)	$\neg\ (/u/=1 \vee /u/=0)$	Hyp. for RAA: non classical u
(2)	$/u/=1 \equiv p$	$(T)_{CL}$, 0, MP
(3)	$/u/=0 \equiv \neg p$	$(F)_{CL}$, 0, MP
(4)	$\neg\ (p \vee \neg p)$	1, 2, 3, subst.
(5)	$\neg p \wedge \neg\neg p$	4, DM

Line (5) is absurd. And this would show the untenability of (1).

However, from our perspective, one could challenge the particular formulation of principle $(T)_{CL}$, specifically its consequent, on the grounds that in order for u to be definitely true it is not sufficient that p obtains, but that p definitely obtains. For instance, the utterance 'Timothy is thin' is definitely true iff Timothy is definitely thin (194-6). So, the replacement of 'true' by 'completely true' in the original (T) requires a change in the right member of the biconditional. Then, what is needed in the consequent of $(T)_{CL}$ is to prefix a 'Δ' to p. Thus, if we restrict our attention to the classical truth values, $(T)_{CL}$ should be reinterpreted as:

$(T^*)_{CL}$ u says that p $\supset$. $/u/=1 \equiv \Delta p$.

The consequent of $(T^{*})_{CL}$ now says that an utterance is definitely true iff what it says definitely obtains. But from here, performing the mentioned steps, we do not get the absurd line (5), but instead:

(5a) $\neg\Delta p \wedge \neg\neg p$.

To obtain (5), we would need to identify "p" with "Δp". And precisely this is what Williamson does. It is worth-while to quote the relevant passage (194):

> Definite truth is supposed to be more than mere truth, and definite falsity more than mere falsity. But what more could it take for an utterance to be definitely true than just for it to be true? ...how could it fail to be definitely true other than by failing to be true?

Williamson here commits himself to maximalism. Hence, an extra assumption is added, which has to be made explicit:

(M) $p \equiv \Delta p$.

(Cfr. 1999a: 129; 1994a: 177; 1992: 150). If this is one premise of the proof, then we can reply that what the reductio demonstrates is not that (1) is unacceptable, but that (M) is so. Indeed (1) is unobjectionable as a characteristic of fuzzy sentences, which are never assigned a classical truth value. But (M) is disastrous for the many-valued enterprise, for it eliminates degrees of truth: truth would be only definite truth. Therefore, when all the suppositions are expressly laid down, the proof does not unequivocally indicate which thesis must be reduced to the absurdum.

A last comment need to be made. Williamson remarks that his reductio «...found the supposition of intermediate cases to be incoherent» (201). If 'incoherent' means absurd, then we do not agree that it has been shown that intermediate cases are absurd. However, if 'incoherent' translates contradictory, then we agree: intermediate cases are contradictory. In fact, from (5a) above, $\neg\Delta p \wedge \neg\neg p$, we can deduce in *Aj*. that:

(5b) $\neg Hp \wedge \neg\neg p$	5a, 'H' for 'Δ'
(5c) $L{\sim}p \wedge Lp$	5b, $\neg Hp \leftrightarrow L{\sim}p$, $Lp \leftrightarrow \neg\neg p$, subst.

The schema "$Lp \leftrightarrow \neg\neg p$" is theorem A152 of *Aj* (Peña 1991: 41). Remember that, when we need to translate the definite operator 'Δ' into the notation of *Aj*, we might symbolize it as the over-affirmation functor 'H', though keeping in mind that they are not equivalent. Formula (5c) says that the situation p partially obtains and partially does not obtain; i.e., p is contradictory, though it does not exhibit the canonical form of a conjunction, one of whose members is the negation of the other. Indeed, a formal contradiction is gotten by means of applying the endorsement or acceptance rule: $Lp \vdash p$. The sentence reporting that borderline fact p will be partially false and partially true. Again that is a contradiction, but it is not impossible.

We can now proceed to see a second interpretation of the proof, a non-classical one. The symbol 'Δ^{+}' will refer to a member of the set of designated truth values, and 'Δ^{-}', to a member of the antidesignated values.

Many-Valued Interpretation

$(PB)_{MVL}$ u says that p $\supset$. $/u/=\Delta^{+} \vee /u/=\Delta^{-}$
$(T)_{MVL}$ u says that p $\supset$. $/u/=\Delta^{+} \leftrightarrow p$
$(F)_{MVL}$ u says that p $\supset$. $/u/=\Delta^{-} \leftrightarrow {\sim}p$

(0)	u says that p	Hyp.
(1)	$\neg$ (/u/=$\Delta^+ \vee$ /u/=Δ^-)	Hyp. for RAA: totally indeterminate u
(2)	/u/=$\Delta^+ \vee$ /u/=Δ^-	$(B)_{MVL}$, 0, MP
(3)	(1) $\wedge$ (2)	(1), (2) adj.
(4)	$\neg\neg$ (/u/=$\Delta^+ \vee$ /u/=Δ^-)	(1)-(3) RAA

Now line 3 is super-contradictory, and this is taken to invalidate the hypothesis (1): if u says something, then to suppose that u is neither designated nor antidesignated is absurd. We totally agree with this proof. Radical indeterminism, which sustains that a fuzzy sentence lacks any alethic status, is incoherent. Yet it is not exactly what Williamson literally says. Additionally, there remains the question of how to justify the first presupposition, $(PB)_{MVL}$.

Thus, in one interpretation of Williamson's proof, if we restrict ourselves to the consideration of the classical truth values, the hypothesis that the fuzzy utterance is neither 1 nor 0 has not been revealed impossible, since we have discovered a tacit assumption, namely, the maximalist thesis, that to be true is to be definitely true, which is the one to be reduced to the absurdum. And in the other interpretation, we have arrived at the result that the hypothesis of an utterance completely lacking in truth value is actually absurd. In brief, in one sense, we accept Williamson's proof, but in another sense, we do not. In any case, we have given different meanings to the original demonstration.

Williamson's argument has been subjected to a critical scrutiny by several authors. It is time to survey the dispute.

OBJECTION 1, by Wright [1994].- Wright examines Williamson's argument that the denial of bivalence is absurd, and he concludes that, for Williamson, no concepts are genuinely vague, for borderline cases, conceived of as a failure of bivalence, have been reduced to the absurdum (135).

Additionally, "Definitely p" and "p" must have different truth conditions. We saw that Williamson equates "p" and "Δp". What is debatable is that p entails Δp, for the inverse entailment may be accepted, that Δp entails p. We will call "p⊃Δp" the *maximalization principle*. Wright (145) indicates that, if there are genuine borderline cases, then the maximalization principle must fail. The proof merely insinuated by Wright may be reconstructed as follows:

(1)	$p \supset \Delta p$	Hyp. for RAA: Maximalization principle
(2)	$\neg\Delta p \wedge \neg\Delta\neg p$	Borderline case
(3)	$\neg\Delta p$	2, Simp.
(4)	$\neg\Delta\neg p$	2, Simp.
(5)	$\neg p$	1, 3, MT
(6)	$\neg\neg p$	1, with $\neg p$ for p, 4, MT
(7)	$\neg p \wedge \neg\neg p$	5, 6, adj.
(8)	$\neg$ ($p \supset \Delta p$)	1-7, RAA

Wright accords the definitely operator, 'Δ', an epistemic interpretation. Its reading would be something like "no one in his right mind could doubt that". More rigorously, that 'p' is *definitely* true means that its opposite assertion, not-p, is cognitively misbegotten, i.e., there is a factor, like defective vision, bad light, etc., explaining why some subject's statement of not-p is in error. Consequently, Wright's conception of borderline cases -line (2)- is that of permissibility of conflict: two competent judges may legitimately differ about a vague sentence. If 'p' is not definitely true, then its negation, ¬p, is not misbegotten, that is, ¬p may be asserted. And if '¬p' is not definitely true, then p should not be misbegotten either and it may be asserted too. Thus, genuine vagueness is a conflict of opinions faultlessly generated, or cognitively un-misbegotten. Wright (146) declares: «So it has to be... *consistent* to suppose either of the parties in... [a] dispute over 'P' to be right...». Therefore, if there is vagueness, it is the maximalization principle that has been reduced to the absurdum.

ANSWER by Williamson [1996b].- Concerning the first criticism that no concepts are genuinely vague, Williamson makes it clear that his position does not deny that our terms are vague. We can say on his behalf that what has been refuted is a particular understanding of borderline cases, but not their existence under any interpretation. Only when we add the controversial assumption that there is no other way to conceive of borderline cases except as a failure of bivalence, then the reductio would show that there is no vagueness. But it is Wright who makes this extra assumption, not Williamson. What the latter tries to do is to provide a hypothesis about the underlying nature of the phenomenon[9].

To the second rebuttal, that, if there are borderline cases, the maximalization principle, "$p \supset \Delta p$", is absurd, Williamson retorts that what is needed in order to set indeterminism apart from agnosticism is a coherent non epistemic interpretation of the 'definite' operator, and therefore, of borderline cases.

OBJECTION 2, by McGee and McLaughlin [1998].- McGee and McLaughlin (M&M from now on) criticize Williamson's contention that the denial of bivalence is absurd. M&M sympathize with a position they call semantic indeterminism, defending a conception of a borderline case as one which the practices and thoughts of the speakers of a language have left unsettled. Thus, a vague word like 'bald' determines a tripartition of men into those who satisfy the predicate, those who satisfy its contradictory, and those who satisfy neither 'bald' nor 'not-bald' (224). So, vague sentences are neither true nor false. M&M think this threefold classification is a logical possibility. However, if Williamson's argument against the repudiation of bivalence were sound, semantic indeterminism would be absurd. This is why M&M attempt to undermine the reductio. Indeed, they call attention to the fact that Williamson defends his disquotational conception of truth and falsity, principles (T) and (F), based on what the words 'true' and 'false' mean. Presumably, (T) and (F) hold because they are analytic assertions. Particularly, what they distrust in Tarski's biconditional is the entailment from the right to the left. That is, the problematic rules used in Williamson's reductio are the T-introduction, and F-introduction:

T-intro: $p \vdash$ 'p' is true;
F-intro: $\neg p \vdash$ 'p' is false.

M&M believe that either Timothy is thin, but the sentence 'Timothy is thin' is not true, or Timothy is not thin but 'Timothy is thin' is not false. Apparently, to understand these possibilities, it is unavoidable to refer to M&M's [1994] article, where they appeal to a many-valued logic and commit themselves «to the existence of degrees intermediate between definite truth and definite falsity» (*Ibid*.: 239), and corresponding intermediate degrees of applicability of the vague term, while at the same time upholding alethic indeterminism: a vague sentence is neither true nor false (*Ibid*.: 214, 221), and rejecting what we are calling maximalism: that for p to be true, it must be definitely true (*Ibid*.: 215, 217). Hence, it seems M&M countenance degrees of truth that are neither true nor definitely true. Consequently, *a* can be *F* to a degree that is insufficient for the sentence '*Fa*' to be considered true, or definitely true. And similarly, *a* can fail to be *F*, but to such an extent that it is not enough for the sentence '*Fa*' to be considered as false. For example, the measure of Tim's waist may be such that, one of the following two alternatives happens: either he is thin, yet the truth value of the sentence 'Tim is thin' is not true, or he is not thin, yet the sentence 'Tim is thin'

9 Crispin Wright, later on (2001: 50), has conceded that Williamson does not deny the existence of vagueness.

is not false. M&M add that this disjunction is true, but not necessarily each disjunct, thereby relinquishing truth-functionality. Therefore, M&M claim that it has not been demonstrated that a tripartite division of the objects in the universe of discourse by the vague predicate is impossible.

ANSWER by Williamson [2004].- Williamson charges M&M with mistaking his conditional principles (T) and (F) for simply disquotational versions thereof, (Td) and (Fd), respectively, to wit:

(Td) An utterance of 'p' is true iff p;
(Fd) An utterance of 'p' is false iff not-p.

To see that these latter formulations are not valid, consider the disquotational principle about meaning:

(Md) An utterance of 'p' means that p.

But the presence of indexicals in an utterance might make it express different propositions depending on the context of utterance. And this is what poses a difficulty for (Md) as is shown by the next counter-example. Your utterance of 'I am not Marcelo Vásconez' does not mean that I am not Marcelo Vásconez. (Td) and (Fd) fail for exactly the same reason. To avoid the problem, the disquotational characterizations of truth and falsity, (Td) and (Fd), should be replaced by the principles (T) and (F), which assert (Td) and (Fd) on the condition that (Md) be satisfied (Cfr. Andjelković and Williamson: 212, 215).

OBJECTION 3, by Simons [1992].- Among other premises, Williamson's argument that the denial of bivalence is absurd uses the Tarski schema:

(T) T('p') iff p,

and the so-called anti-bivalence formula:

(∼PB) ∼T('p') ∧ ∼T('∼p').

Simons' main line of attack consists in signalling an ambiguity in the symbol 'T'. It may mean either truth or clear truth. In the first sense, when 'T' means simply true, the (T) schema can be accepted, but the (∼PB) is unacceptable. Indeed, accepting (T) means that truth is disquotational or redundant; and, for a vague sentence 'p', if the right member of (T) is indefinite, then its left member is indefinite too, for T('p') takes whatever value 'p' gets, but the biconditional is true (172). Yet (∼PB) fails, since neither conjunct is true, independently of which negation one uses, either De Morgan or Boolean. On the other hand, if 'true' means clearly true, then the opposite happens, that is: (T) fails for clearly true, but (∼PB) is valid when it is applied to clear truth. In fact, an instance of (T) is:

'Frank is bald' is clearly true iff Frank is bald[10].

If Frank is a borderline case, then the left member is false, while the right member is neither true nor false. So, the members of the biconditional do not have the same truth value. However, the (∼PB) is valid for, typically, neither p nor its negation are clearly true.

10 Notice that Simons omits the adverb 'clearly' on the right hand of the biconditional, and for that reason we may object that the sentence is not a proper instance of the Tarski schema.

Therefore, Williamson's reasoning equivocates, having true premises only if we shift the meaning of the word 'true' from one premise to the other. What he has at most shown is that the sum of intuitions about truth is incoherent.

It should be added that Simons does not consider the Tarskian schema as an analytic property of truth, i.e., as a *sine qua non* condition of any concept of truth.

ANSWER by Williamson [1992].- Because Williamson's [1992] article was not strictly speaking a reply to Simons, I conjecture that part of its response would be that, in the proof that non-bivalent conceptions of vagueness are not coherent, it is supposed that the Tarskian schema, despite its not exhausting the concept of truth, is indeed an essential part of it (1992: 148, n. 6). What else could it take for 'Tim is thin' to be true than just for Tim to be thin?

3e.- Continuing the Debate

We now proceed to review other important issues in the rich exchange of ideas between Williamson and his critics. It is hoped that we will gain some enlightenment by realizing both, which points have been subjected to doubt, as well as how Williamson has buttressed them up. Necessarily, we have to leave many details aside. For example, we omit reference to critical commentaries by Delia Graff [2002a] and Mario Gómez-Torrente, due to their highly technical nature, though their remarks, together with Williamson's reply [2002a], have been taken into account.

3e.i.- With Crispin Wright

I want to highlight basically three criticisms made to Williamson by Wright [1994], which deal with the following topics: i) the existence of sharp cut-off points; ii) the unknowability of sharp boundaries; and iii) deliberate approximations. Let us see each objection in turn. But before that, it is fair to record that Williamson's merit, as recognized by Wright, is his bringing out that the idea of semantic indeterminacy is not a datum.

First, we should inquire whether Williamson has managed to make some sort of case in favour of the existence of a clear-cut border in the meaning of vague words. This question is fundamental, and it should be treated with special care. Remember that Williamson (p. 1) has claimed that, if the principle of bivalence held, then the soritical series would be bipartitioned. Thus, for example, there would be a last second in which Rembrandt is not old, immediately followed by a first second in which he is old. So, sharp boundaries would follow from bivalence.

In this connection, we record Wright's opposition to the existence of a neat cut in the soritical series. He rightly contends that, since the application of a colour predicate, '*F*', depends on observation alone, it is unjustified that we attribute *F* to only one of two contiguous neighbours, for we cannot perceive any difference between them. He affirms that:

> it is... absurd to... justify incompatible colour judgements about items that look exactly alike (151).

Second, concerning the demonstration of why we cannot know the exact location of the sharp cut-off points, Wright says that Williamson has not proved the absolute impossibility of knowing the borders but only the relative unknowability given the methods that we normally use for the application of vague predicates. For illustrative purposes, consider a series of four hundred canes ranging from 1.6 m to 2 m in height and such that each is 0.1 mm higher than the preceding one. Now by visual means alone, and without doing any sort of calculation, I cannot know which canes are at least 1.75 m high, due to the MfE principle that Williamson formulates, for, although I may have managed to pick up the correct subset of canes, that would be a matter of good fortune rather than of reliability of the belief forming mechanism; i.e., I could too easily have made the selection which included the cane of height

1.749 m, and thus I could have had the wrong belief. So, in these conditions, I would be prevented from having knowledge of which sticks are at least 1.75 m high. However, this does not eliminate the possibility of acquiring that knowledge by others means; concretely with the help of a measuring instrument. Thus, what Williamson has shown at most is that we cannot have knowledge of the cut-off point when we restrict our consideration to unaided perception, which is the ordinary method for application of vague expressions; but he has not demonstrated the impossibility of that knowledge *tout court*.

Furthermore, besides proving that no one can know where the borderline is, Williamson should establish that we cannot know where the frontier is not; i.e., that it is not possible for us to discover that both x and x' are *F*, nor that neither x nor x' are *F*, for a couple of very similar members of the soritical series. The reason why this extra proof is needed is not explicitly stated in Wright's article (149-50). However, I surmise that Williamson must counter an argument to the opposite conclusion that we can know that there is no boundary there where Williamson thinks there is one. We can reason to this cognitivist conclusion by following the pattern: Kp, K(p⊃q) ⊢ Kq. Indeed, letting p' be as before, the sentence describing the situation which minimally differs from the one described by p, and assuming that we know the similarity principle, $(SP)_{CL}$: p∧p' ∨. ¬p∧¬p', and that we know that the (SP) entails in CL the continuation principle, $(CP)_{CL}$: ¬(p∧¬p'), then we can know the (CP), which contradicts the agnosticist's claim. Thus, Williamson would need to show that we cannot know the (SP), i.e., that we cannot know neither p∧p' nor ¬p∧¬p'. Yet, Wright claims that, from the MfE principle, there is no way to deduce that (p and p') is unknowable, nor that (neither p nor p') is unknowable.

Third, Wright alludes to expressions in English which deliberately introduce vagueness, like 'approximately', 'roughly', 'about', 'almost', etc. which ensue in indefinitely expanding the set of truth conditions of the sentence in which they appear, making it true in an unspecified wider number of cases. Wright challenges Williamson to explain how the flexibilization of truth conditions is effected if those words have a sharp meaning. The agnosticist may specify an exact range of wider truth conditions making the vague utterance true, but the problem is that how that widened precise set of truth conditions fit the intention of the speaker that an approximate set of truth conditions will be good enough.

Surprisingly, in the article [1996b] devoted exclusively to reply to Wright, Williamson has not answered to all the charges made. Particularly, no answer is to be found to: 1) an aspect of the second objection, namely that no proof is offered of the impossibility of knowing that both x and x' are *F*, or of the impossibility of knowing that neither x nor x' are *F*; 2) the third challenge, that of explaining the relaxation of the truth conditions by means of a subclass of vague words.

As for the second objection, that there are other ways to know whether a borderline case is *F* other than by unaided observation, Williamson says that the property of measuring at least 1.75 m is precise, and thus it can be determined using a measuring tape. But in the case of a vague property, like tall, the appeal to a metre is out of place for there is no definition available of vague words in terms of exact ones. So, there is no procedure that will allow us to know whether somebody is tall when the person is borderline. To change the example, we have no idea as to how to come to ascertain the exact minimal number of grains constituting a heap. This is the datum of our inquiry; and agnosticism explains why this is so (41).

3e.ii.- With Machina and Deutsch

The whole effort displayed by Machina and Deutsch [2002] may be seen as aimed at disproving Williamson's claim that vague predicates have precisely bounded extensions. They canvass several attempts at justifying this claim.

They adopt a non-bivalent perspective of vague predicates. The critics assert that there is agreement in considering that, in borderline cases, we do not know what to say, or that there is irremediable uncertainty as to whether the predicate applies or not. What explains this uncertainty is that vague predicates, as a matter of their very meaning, do not have precise boundaries; i.e., their definition makes it impossible to specify a precise cut-off point marking the exact extension of the predicate. Thus, any definition of a vague predicate consisting in a precisification will necessarily be incorrect, since it will not preserve the term's vagueness, making it precise. If this is so, then it follows that Williamson's theory denies the existence of vague predicates, since it only allows precise predicates with precise extensions, a conclusion similar to that reached by Wright. But has the existence of precise extensions for vague predicates been proved? They contend that there is no satisfying reason for this.

First, we saw that Williamson's reason supporting the existence of sharply delimited extensions is that Classical Logic requires them. To this Machina and Deutsch reply that a logic of vague predicates may enlarge and develop CL, by allowing non-bivalent predications. And it constitutes no objection against such a logic the fact that it restricts the validity of classical logic to the cases where no vagueness is involved, in the same manner as the theory of relativity is not refused just because it makes classical physics have a limited range of applicability.

Second, there is linguistic evidence that certain vague expressions cannot have crisp limits. For example, the predicate 'somewhat tall' does not signal a hidden but precise border, but, on the contrary, suggests an inexact boundary.

In his rejoinder [2002c], Williamson disapproves of Machina and Deutsch's characterization of his position as one holding that a vague predicate has a precise boundary. Since 'vague' and 'precise' normally are contrary terms, the critics would impute an incongruity to agnosticism. To remove this terminological problem, a better denomination is 'excluded middle boundary', in the sense that everything falls either inside the boundary of the predicate or outside it. Thus understood, it is true that, for the agnosticist, vague predicates have excluded middle boundaries.

Moreover, Williamson complains about Machina and Deutsch's failing to address the objections he has levelled against degree theories. In fact, in his *Vagueness*, he has tried to show that non classical conceptions are not illuminating, and do not provide an account of the phenomenon, especially higher order vagueness[11].

Again, it is worth noting Williamson's silence with respect to the semantics of the subclass of vague words typified by 'somewhat'. As Wright has also suggested, there will be a mismatch between precise, bivalent truth conditions and vague meaning.

3e.iii.- With Stephen Schiffer

Stephen Schiffer ([1997], [1999]) believes that the greatest obligation of agnosticism is to explain several kinds of ignorance to which it is committed. We will concentrate on these troublesome obligations in the present sub-section.

But first, there is a preliminary disagreement concerning the bearer of truth, and consequently, the proper formulation of the principle of bivalence. Schiffer thinks that what is true or false is a proposition[12] (the content of an utterance). A first reason for this tenet is that it is consistent to maintain the following two theses. (a) A supervaluationist conception of propositions, according to which the borderline proposition that Harry is bald and its

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11 For a description of Williamson's charges against a many-valued approach, and my answers to them, see Chapter 4, section 7.

12 Schiffer (1999: 485) defines *proposition* as an ordered pair of the form $<<x_1, ..., x_n>, F^n>$, where '$<x_1, ..., x_n>$' is a sequence of n objects and 'F^n' denotes a n-ary property. Such proposition is true iff the sequence instantiates the property, and it is false iff $<x_1, ..., x_n>$ does not instantiate F^n.

negation, that Harry is not bald, both lack a truth value, even if the disjunctive proposition that Harry is bald or not bald is true. (b) A deflationist or disquotational account of truth for utterances, such that what is said by 'the utterance *u* is true' is the same as what is said by the utterance *u*; and what is said by '*u* is false' is the same as what is said by uttering the negation of the utterance *u*. If this second thesis holds, then, when one affirms that the borderline utterance 'Harry is bald' is either true or false, one is just uttering Harry is bald or not bald. Anyone accepting classical logic, can trivially have the principle of bivalence by appealing to a disquotational sense of 'true' and 'false'. Thus, for the theorist upholding (a) and (b), the principle of bivalence applies to utterances, but not to propositions. The second motivation to support that the truth bearers are propositions is that Schiffer thinks that a conception of vagueness as ignorance must be formulated in terms of that-clauses, because, in general, knowledge is analytically tied up with propositions, and, particularly in our discussion, if Harry is a borderline case of baldness, then agnosticism maintains that we ignore that Harry is bald, we do not know the truth value of what is said.

Hence, the definition of agnosticism will comprise three elements: that there are vague propositions, for which bivalence holds, and that we are ignorant of which truth value they have.

According to Schiffer, agnosticism encounters its major difficulties at the moment of explaining three kinds of impossible knowledge which it endorses, namely: ignorance with respect to which truth value a borderline proposition has; ignorance of the necessary and sufficient conditions -let us call them *G*- for being *F*; and ignorance that the word '*F*' means the property *G*. To continue with our previous example, Williamson has to discharge the debt of accounting for three items: first, our inability to know that Harry is bald, when he is a borderline case of baldness; second, the impossibility of knowing the necessary and sufficient conditions *G* for a person to be bald, say, to instantiate the property of having less than 3,832 hairs on her scalp; and third, the principled ignorance that the adjective 'bald' means the precisely delimited property *G*.

Schiffer claims that Williamson's explanation of our inability to know the truth value of the vague proposition does not require appealing to the notion of inexact knowledge nor to the principle of margin for error, for the reliability of knowledge suffices. The account goes like this. Suppose that Harry has exactly 3,831 hairs on his scalp, and that Jane has the true belief that Harry is bald. Now, due to a slight change in the conceptual role of her concept 'bald', this concept may come to mean a slightly different property, namely, baldness*, which -let us suppose- is the property of having less than 3,831 hairs. But, since this semantic change can pass unnoticed to Jane, her belief forming mechanism would all the same put the belief that Harry is bald in her belief box even though that belief is now false, due to the change in meaning of the concept 'bald'. But, if a mechanism produces false beliefs as easily as it produces true ones, then it is not reliable. Precisely this is what happens in our case. And because the mechanism is unreliable, the initial true belief that Harry is bald cannot qualify as knowledge. Or, in other words, we cannot know whether it is true that Harry is bald. Q.E.D. Thus, since the notion of reliable knowledge has done all the job, the idea of inexact knowledge and the principle of margin for error are rendered unnecessary.

Now let us suppose that the following data are available. (a) That there is a property *G* such that: Harry has *G*; Jane can know that Harry has *G*; that any person possessing *G* is bald, i.e., that *G* is metaphysically necessary and sufficient for being bald. (b) That *G* is the property of having less than 3,832 hairs on one's scalp. (c) That Harry has exactly 3,831 hairs on his scalp. And (d), that the fact that 'bald' means *G* supervenes on facts of the relevant sort about use plus environment. Given all these data, one would expect that Jane can know that a subject is bald whenever he has less than 3,832 hairs, and therefore Jane can know that Harry is bald. Williamson denies this. If we ask why one cannot know that '*F*' refers to *G*, Williamson will answer that it is impossible to know that because of the vague-ness of the reference relation; i.e., we are prevented from having knowledge of which exact property 'bald' refers to due to our limited capacity to discriminate closely resembling properties.

The most serious problem for agnosticism is that it seems to make communication impossible. For, if vague vocabulary has such a sensitivity to the slightest change in use, then no two persons belonging to the same linguistic community could mean the same thing by the same word, in view of the different personal use of the expression in question. When you say 'Elizabeth stood roughly there', the complement of place denotes a precisely delimited region, but I have no idea of what that is. It would be a miracle if your use of 'roughly there' exactly matches mine; consequently, the utterance would be understood differently by speaker and hearer, and it is not clear how they can communicate.

Williamson, in [1997c] and [1999b], has replied to both articles by Schiffer.

A first point inviting a commentary concerns the question of whether agnosticists should formulate their theory in terms of propositions, as truth bearers. Williamson feels qualms about Schiffer's use of propositions, especially because the latter has not clarified how quantification over propositions operates. Nonetheless, Williamson concedes that, if it is assumed that an utterance expresses a unique proposition, then epistemicism can be defined in the way Schiffer recommends, as the position sustaining that there are vague bivalent propositions whose truth value is unknowable. From this alternative formulation, Williamson infers that the definition of 'epistemicism' is vague, in the sense that there is no set of necessary and sufficient conditions for its application.

As for the difficulty that agnosticism makes communication impossible, given that -leaving the influence of the environment constant- a slight change in use produces a difference in meaning, Williamson replies several things. First, that the problem of reconciling difference of use with sameness of meaning is not exclusive of agnosticism, but equally confronts any reasonable theory of vagueness. Second, that the reference for me of your utterance of 'a little while' in certain context may be fixed parasitically by whatever it is that it refers to for you. This is what Williamson calls '*deferential reference*', which is similar to reference borrowing. Third, he says that understanding between speaker and hearer for most practical purposes does not require perfect matching of personal dispositions to use a word, for it is sufficient that both mean «roughly the same» thing. Of course, communication would fail if both had radically different uses; but a slightly different use does not preclude communication (1997c: 952-53). Fourth, Schiffer has not proved that we cannot direct our attention to a region of space when we lack knowledge of its precise limits. In general, in order to know the meaning of a word it is not necessary to know its sharp boundaries (1994b: 211).

3e.iv.- With Peter Mott

Mott [1998] has taken issue with Williamson about the margin for error principle (MfEP). He mentions two problems with it, but before considering these, let us define what the *personal margin for error*, *i*, is. Suppose people judge the height of trees by eye. If Bob's margin for error is 1.5 m, and he estimates the tree is n metres high, then: (a) the tree is n $\pm$ 1.5 m high; (b) Bob knows that the tree is between n+1.5 and n-1.5 m high; and (c) there is no number smaller than 1.5 m satisfying the previous two conditions. That is, given the value of *i* for a certain subject who makes an estimation, we can derive consequences regarding the fact and the knowledge related to that estimation. For example, if Bob estimates the tree is 10 metres high, then the tree's height in fact falls in the interval from 8.5 to 11.5 m, and he knows that the tree is at least 8.5 and at most 11.5 m high. We have to bear in mind that a belief of the tree's height, h, is knowledge only if the estimated tree's height falls within the interval h $\pm$ *i*, delimited by the personal MfE.

Now, the first difficulty with Williamson's MfEP concerns the knowledge of other people's knowledge. Consider three friends: Steven, Anna and John, standing in front of a tree 7 metres high. Let us assign all of them a margin for error of 1.5 m Steven says that "the tree is no more than 10 m high", which we will symbolize as 'p'. So, he knows that p. Anna, making her own estimation, agrees with him, and says that Steven is right in judging the tree no more than 10 m high. We may correctly say that she knows that he knows p. And finally, John, witnessing the previous conversation, comes to voice his opinion that Anna knows that Steven knows that p. We may assume that John knows that Anna knows that Steven knows that p. However, this assumption is disallowed by Williamson's MfEP. To prove this, Mott uses a MfE principle having to do with the reiteration of the knowledge operator, namely,

$$K^n \neg \text{ (tree > m metres high)} \supset \neg \text{ (tree > m-(}i \times \text{n) metres high)},$$

where '*i*' is the accuracy for the margin for error, and 'n' the number of times the operator 'K' is reiterated.[13] The effect of this principle is that each time 'K' is applied, the height of the tree diminishes in the consequent of the principle. For example, for the first two friends, Steven and Anna, their respective MfEP is:

$$K_s \neg \text{ (tree > 10)} \supset \neg \text{ (tree > 8.5)}$$
$$K_a K_s \neg \text{ (tree > 10)} \supset \neg \text{ (tree > 7)}$$

Both conditionals are true. If Steven knows that the tree is no more than 10 m high, then the tree is no more than 8.5 m high, because the latter height is within the MfE for Steven, i.e, a variation in the objective circumstances of at most the value of *i* is supported by the MfEP. When Anna utters her belief, she brings in her own MfE; this is why the height of the tree goes down to 7 m high, which is still right. However, the corresponding principle for John fails, since we have that:

$$K_j K_a K_s \neg \text{ (tree > 10)} \supset \neg \text{ (tree > 5.5)},$$

but we are supposing that the tree is 7 m high. Mott concludes that, the MfEP is too restrictive of our limits of knowledge.

Mott next tries to refute the MfEP by attempting to prove that we can get the knowledge that the principle takes away from us. Remember that according to Williamson, it is not possible to have knowledge of an object x that is very close to the borderline of *F*, for if x is *F*, it might become not-*F* by a minute change, invisible to us, so that we might wrongly believe that x is *F* when it is not. For example, when the actual number of people present in a stadium is 18,473, it is not possible to know, by a rough estimate, that there are not 18,474 people because a margin of error of one head cannot be credited to any person in this kind of inexact knowledge. For agnosticism, we could have that knowledge only if we could judge the number of actual people in the stadium very accurately. Mott wants to demonstrate that this is mistaken; that we can have knowledge even though we start from an inaccurate estimate.

Suppose again that the personal margin for error, *i*, of Tom at estimating trees is 1.5 m. Anew, we are going to exploit the fact that a variation in the height of the tree of at most the value of *i* is permitted by the MfEP. Thus, Mott's goal is to falsify the agnosticist statement that:

$$\text{tree} = 7.25 \supset \neg K \neg \text{ (tree} = 7)$$

That is, if the tree is 7.25 m high, then it is not known that the tree is not 7 m high. This is just the contraposition of an instance of the MfEP:

13 Williamson will agree with this consequence of his MfEP. He says: «...knowledge that one knows requires two margins for error. More generally, every iteration of knowledge widens the required margin» (1994b: 228).

$$K \neg (tree = 7) \supset \neg (tree = 7.25)$$

If it is known that the tree is not 7 m high, then the tree is not 7.25 m high. Now, assume that Tom estimates that the tree is 8.75 m high. Then, by the definition given above of the personal MfE, he knows that the tree is 8.75 ± 1.5 m high, i.e., he knows that the range of the tree's height is [7.25, 10.25]. Therefore, by a simple deduction, he knows that the tree is not 7 m high, contradicting the result of the MfEP.

Of course, Mott (500) is aware that for each person there is no unique value *i* for her MfE. Nonetheless, he asserts that different values of the size of *i* will not affect the argument (501).

As a general conclusion, Mott claims that the MfEP is systematically wrong, that it is not a good basis for a conception of inexact knowledge, nor for Williamson's theory of vagueness.

Williamson convincingly replies to Mott in his [2000]. The main mistake incurred by Mott in both of his arguments is that he neglects the inexactness of our knowledge of inexact knowledge. To assume that we have precise knowledge of our own margin for error, *i*, or of that of other's, is not warranted. No experiments can help us in determining the exact value of our MfE. Mott supposes that the same *i* is applicable to the three friends.

(1) If Steven, Anna, or John knows that the tree is n metres high, then it is n-1.5 m high.

But the estimate that Steven makes is based solely on his visual perception, whereas those of Anna and John are influenced by what has been said before in the conversation. This means that John's MfE may be smaller than Anna's MfE, which in turn may be smaller than Steven's. Furthermore, even if Anna knows (1), nothing guarantees that John knows this fact that Anna knows (1): John may be ignorant of how well Anna is acquainted with Steven. So, to presuppose (1) while arguing against MfEP is illegitimate.

Further, consider the first definitional characteristic of the personal margin for error given by Mott.

(2) If I estimate the tree is n metres high, then the tree is n±*i* m high.

Williamson says that I cannot know (2) if it is too close to a situation in which (2) is false. An instance of (2) is:

(2') if I estimate the tree is 15 m high, then the tree is 15 ± 1.5 m high.

But it may well be the case that if I estimate the tree is 15 m high, then the tree is 15 ± 1.4 m high. I cannot know (2) because I do not know how good an estimator I am.

So, Williamson concludes that no good reasons have been presented against the claim that the MfEP expresses a necessary condition for knowledge.

3e.v.- With Peter Simons

We now examine the opinions of Simons, in [1992] and [1996], on vagueness and ignorance, in order to further contrast Williamson's views.

One point of disagreement concerns the nature of vagueness. The principle of bivalence affirms the existence of only two truth values, True and False (1992: 163), and mandates that every sentence have just one of them, prohibiting that it have neither and that it have both. However, Simons thinks that this principle is indeed threatened by vagueness since this is characterized as the possibility of having borderline cases, which in turn are indeterminate. Utterances expressing borderline cases are neither true nor false (*Ibid.*: 172).

Both, bivalence and also the principle of excluded middle fail (1997: 307). There is a semantics capturing at least part of vagueness that rejects the principle of bivalence (1995: 202).

Moreover, Simons claims that the uncertainty of borderline cases is not merely epistemic, but semantic. Take for example the colour of the marble used in public buildings in Salzburg, being brownish red or reddish brown. If it is really a borderline case seating firmly mid-way between red and brown, then there will be no discovery that we could make helping us to determine the question whether the marble is red or brown. There is no fact of the matter, no hidden fact of which we are ignorant. Simons insists that whenever we have no way to get information to answer an inquiry, we believe that there is nothing out there that could settle the issue.

Simons' article was preceded in the same volume by Williamson's statement of epistemicism [1992], which was later developed in his classic *Vagueness* [1994b]. Though the primary purpose of Williamson's [1992] article is to give a critical presentation of his own agnosticist position, we might extract some ideas that are apt to be seen as answers to Simons' worries.

The nature of vagueness is inextricably connected to bivalence and to our ignorance. The only sense in which vague sentences are neither definitely true nor definitely false is that they are not knowably true nor knowably false (1992: 151). Yet Williamson clearly states that the principle of bivalence has nothing to do with the definite operator. The PB does not mention the latter.

3f.- Summary

We have finished examining the objections to agnosticism and the corresponding replies to them by Williamson. As a summary, I want to highlight Williamson's recognition that the cutoff point marking the division between two contradictory fuzzy properties is not fixed by nature (1994a: 183). That is, in reality there is no discontinuity. Now, if the world does not support any crisp borderline, then it is understandable that Williamson has to treat fuzziness as somehow arising from the subject. For him, fuzziness has a subjective origin, it is «constitutively dependent on thinker's epistemological limitations» (2003: 712). However, our general, strongly realist orientation pushes us in the opposite direction. We should avoid positing any sharp boundaries, to do justice to the continuity found in reality. Furthermore, there are fuzzy expressions that resist any attempt to be treated as if they had finely tuned borders, such as "somewhat tall", "approximately two kilometres", "more or less blue", etc. We should continue our search for a better theory of fuzziness, one that respects fuzzy borders.

4.- Sorensen's Agnosticism

Roy Sorensen is another agnosticist active in the debate. However, his position is a peculiar one, since he also defends that competence in English forces on us inconsistent beliefs. In this section, I will focus on his book *Vagueness and Contradiction* [2001].

As before with his agnostic brother Williamson, a vague sentence is thought to have one of the two classical truth values, though it cannot be known which value it actually has. Sorensen is thus also opposed to any vindication of objective indeterminacy.

In order to differentiate himself from Williamson's agnosticism, Sorensen distinguishes two sorts of borderline cases: relative and absolute. The former involve ignorance relative to a cognizer; the undecidability varies with the subject, or depends on the methods or procedures used. Sorensen claims that Williamson's borderline cases are relative to human beings, for what is indiscriminable for us may not be so for a creature with greater discriminatory power (177; Williamson 1994b: 212). Furthermore, Williamson's explanation of ignorance makes reference to the thinker (14). Indeed, Sorensen accuses Williamson of anthropocentrism, since the latter takes human unknowability as the standard of borderline cases (48). But vagueness has to do only with absolute borderline cases, those whose uncertainty cannot be removed in any manner whatsoever.

On the other hand, vagueness is a kind of subjective phenomenon, since all the dilemmas posed by vagueness and the sorites spring from our system of representation rather than from the world itself. In a sense, those problems are the products of our own making.

As for the sorites, the agnosticist solution consists in upholding the validity of the logic of the reasoning, and in rejecting one instance of the conditional major premise. Sorensen indicates that a supplemental logic, one that only augments the stock of classical theorems or inference rules, cannot invalidate the form of the sorites. It will be a deviant logic the one that takes the step of renouncing the validity of the inference rule of the sorites. But we should not change the (standard) logic, Sorensen contends, to rescue a speculative hypothesis about how language works. Moreover, no deviant logic is accompanied by a single success. He means that while Classical Logic has made possible the landing of man on the surface of the moon and countless other technical achievements, nothing comparable occurs based on non-standard logics.

Given the adherence to CL, the only remaining plausible way to avoid the absurd conclusion is to deny the major premise. Hence, Sorensen also accepts the negation of the premise, which implies that there is a number of grains, n, such that n grains constitute a heap, but that n-1 is not a heap. So, there is a sharp threshold point at which an eroding heap becomes a non heap. The word 'heap' is sensitive to the removal of a single grain.

In order to dismiss the charge of arbitrariness in the assignment of contradictory predicates to contiguous members in the soritical series, Sorensen renounces the *Truth Maker Principle*, which requires that each truth be made true by some fact in the world. Sorensen is of the opinion that this principle is an overreaction to the correct tenet that truth supervenes on being, instead of floating in the void. In the absence of the truth maker principle, a borderline sentence can receive a truth value without being prescribed by the world. Concretely, in the case of the conjunction stating the occurrence of the sharp threshold, $Fa_i \wedge \neg Fa_{i-1}$, the invalidity of the truth maker principle robs us of any rationale to expect that there is something in the world that makes a_i the last F, while making a_{i-1} the first not-F.

On the other hand, it is the failure of the truth maker principle that provides a universal account of the ignorance of absolute borderline cases (177-78). For, if the sentence's truth is not grounded in any fact whatsoever, but it is rather autonomous, like something epistemically isolated not resting on anything else, then there is nothing that could serve as evidence of its truth, nothing that could function as its supervenience basis. Then, any cognizer is at a lost, not having any means that could help her to track the truth value. From this, it results the impossibility of an explanation of why the threshold is located where it is.

Despite the falsity of the major premise, there are also several reasons advanced by Sorensen to regard that premise as true. First, it is not possible to find a counter-example to it (2), and though it can be false, it cannot be shown to be false, and in this sense, it is *incorrigible*, i.e. «no one is ever in a position to *show* that the statement is mistaken» (59). Second, the major premise is not only *analytically true* (63), but also *a priori true* (108), that is, true by the meaning of the words, and independently of any empirical investigation, respectively. The reason for the apriority of the premise is that everyone has a right and a duty to ignore insignificant differences among adjacent members in the soritical series. Third, the conditional premise must be true since Sorensen expresses a desire to maintain the norm that like cases should be treated alike (44). The indiscriminability is so much fine-grained that the negation of the conditional major premise would be contradictory (108). So, there are grounds to consider the major premise not only as not false, but also as positively true.

Thus, given that both the major premise and its negation are incorrigibly believed (59) and both appear to be a priori true (108), Sorensen resolutely embraces the thinkability of contradictions, that is, it is possible to think the impossible, $\diamond\exists p\exists x\,(\neg\diamond p \wedge Bxp)$, where 'x' stands for a believer, and 'B' for the belief relation with respect to a proposition "p". He wants to dispel the impression that the cure is worse than the disease, as if one said: «The good news is that the sorites paradox has been solved. The bad news is that the solution comes at the price of believing infinitely many contradictions» (20). Of course, it is not that bad. The belief in contradictions is demanded by reason (20); it is inescapable (57, 60). They are rationally mandatory (91). On the one hand, pairs of contiguous members in the soritical series are indistinguishable for a subject, due to the fineness of the indiscriminability among them. But on the other hand, the extremes, a_0 and a_z, are well differentiated. So, in a sense, the subject 'sees' that the three elements of the inconsistency are each true: Fa_0, $Fa_i \supset Fa_{i+1}$, $\neg Fa_z$ (83). Though English itself is not inconsistent, competence in English compels belief in many contradictions (19).

Then, an inconsistency is possible; but this happens solely at the level of beliefs. Sorensen is far from being a supporter of paraconsistency. To handle inconsistent beliefs it is not necessary to alter CL. While a system of beliefs cannot be totally free of contradictions, a contradiction in itself cannot be both true and false, on pain of using the word in a deviant sense (147). The meaning of a contradiction is inextricably tied up to its role in the reductio ad absurdum reasoning: once it is shown that p entails q and not q, then one thereby rejects p (77). If a contradiction is detected, it is immediately abandoned, because nobody can believe what she regards as false (155). If there is a way to add p and not-p to a system without inconsistency, that can only be at the cost of incompleteness. «If I perceive a proposition as a contradiction, then I cannot conceive of how it could be true or how there could be the least bit of evidence in its favour» (108). This is what Sorensen says of the negation of major premise, which is taken as a contradiction. Again, he asserts that it is irrational to believe p after you concede that it has nothing more in its favour than not-p (28).

Let us summarize the points made. Though the conditional premise cannot be contradictory, in the sense of being both true and false at the same time, we have basis to believe that it is true and false. It is not impossible for us to believe in a contradiction. Rather, competent speakers are forced to believe in contradictions.

4a.- Assessment

We cannot but agree with the last conclusion reached. Yet, we think that contradictions should not be relegated only to the realm of our beliefs. Sentences can rationally be true and false. Sorensen has put forward reasons to believe that the major premise is true and false. If we grant they are good reasons, then, the major premise is true and false.

5.- Conclusion

There are obvious points of (partial) contact between agnosticism and our own position. I mention the most important ones. First, I share the need to conserve classical logic in its entirety. This means that it is not necessary to drop any tautology or theorem, or inference rule of CL in order to cope with fuzziness and the sorites paradox. This, of course, should be understood with the proviso that we read the standard negation as a strong one. If this is so, then indeed, fuzziness does not require to withdraw any classically valid truth nor rule of inference, but it does call for an addition to CL. Besides, we think agnosticism is also correct in its demand that we keep the principle of bivalence, on the condition that PB be taken as requiring no more than that every sentence be true or false, which should not be confused with the need for attributing to each sentence one of the classical truth values, True or False (1 or 0). Consequently, we join agnosticists in their distrust of pure indeterminism.

There are also smaller points of (qualified) agreement, such as the belief that fuzzy words are sensitive to minute changes. For example, the removal of a single grain may change the truth value of the sentence 'a_i is a heap'.

On the negative side, the three authors, Quine, Williamson (2003: 706) and Sorensen, all renounce the truth maker principle, and the major premise of the sorites. We believe that these losses are serious, as it was shown in Chapter 1, section 5. See also Chapter 6, section 5, for a general criticism.

CHAPTER 3

INDETERMINISM AND SUPERVALUATIONISM

In the last chapter, we mainly exposed a theory that sees vagueness and the sorites as posing absolutely no threat to the universal rule of classical logic. Now in this third chapter, we introduce a different sort of outlook, one that finds fuzziness and the heap paradox as a reason to be dissatisfied with some aspect or other of classical logic. We present the indeterminist views of Bertrand Russell and Michael Tye -though the former has also some elements of agnosticism-, and the supervaluationist conception of Kit Fine and its defence by Rosanna Keefe.

1.- Russell

Bertrand Russell published an article on vagueness in 1923. His purpose was to prove that all languages were vague. That is, not only common names, like "red", or the notions employed in the sciences, but also proper names and even the vocabulary of logic are all infected with vagueness. And this is not a peculiarity of English, but a feature shared with other tongues. The reason why logical words, like "or" and "not", are said to be vague is that the words by means of which they are explained, namely "truth" and "falsehood", are also vague, since these in turn cannot be defined in precise terms (64-65).

Russell takes vagueness to be a characteristic of meaning, or, in general, of the relation between the representing system and what is represented. There is vagueness when this relation is one-many, in opposition to what is an exact representation. For example, when to a word there correspond several objects, or when many possible facts may verify a sentence. It is for this conception of meaning, as a one-many relation, that he has been seen as the forerunner of supervaluationism (Cfr. Hyde 1992).

The source of vagueness for Russell lies in our limited capacity of perception, i.e., in the fact that there are stimuli whose content is different but cause the same sensation in us. Vagueness appears as a result of our deficient power of sensual discrimination. He then believes that all sensible words are vague, in that we have the same word for different things.

What is of particular interest for us is Russell's insistence that there is vagueness or precision only in our opinions, knowledge or language, but not in the things or in the world. If we applied vagueness to reality, we would be committing the *fallacy of verbalism*, that is, we would mistake a property of words for a property of things. His ground for attributing a logical error to the friend of ontological vagueness is Russell's belief that

> ...things are what they are, and there is an end of it. Nothing is more or less what it is, or to a certain extent possessed of the properties which it possesses (62).

We take this quotation as a conspicuous pronouncement of *maximalism*; that is, things possess their properties absolutely, without qualification, but not in degrees. One possible ground for this exclusion -I venture to guess- is that, if we admit gradual attributions, we open the door to contradictions. For example, eleven karat gold is partially gold, but rather it is not gold; hence it is and it is not gold. The first sentence of the quote seems to express Russell's rejection of contradictions, rather than a vacuous and trivial tautology. Things are what they are, and -we might supplement- they are not what they are not. Gold is gold. If an alloy has less than 50% gold, then it is simply and plainly not gold. Thus, Russell's vision of fuzziness appears to be rooted in his dislike of degrees and contradictions.

Nonetheless, in the next two paragraphs immediately after the passage quoted above, Russell also gives hints of an awareness of an objective graduality. He makes the following three points. (a) He literally says that processes like the birth and the death of a person -say Peter- are «gradual». He further contends that: «If you continue to apply the name ['Peter'] to the corpse, there must gradually come a stage in decomposition when the name ceases to be attributable...» (63). (b) Russell asserts that it is obvious that colours form a continuum. And hence, finally, (c) he thinks that the existence of a hair which turns a hairy man bald is absurd. Notwithstanding, since the tenor of the commanding first quotation (62) is antigradualist, we cannot interpret his subsequent words in the sense of a gradual ontology. In effect, I suspect that when Russell associates graduality with doubt, uncertainty, and indecision, he is making a reduction of the talk of degrees to something subjective. And when this is not the case, then he brings indeterminacy into play.

Thus, by (a) he probably means nothing more than that nobody knows precisely when the name of the person begins to be attributable and when it stops being so. For sure, there are stages at which the name is certainly applicable, and others where it unquestionably is not so. But between these two extremes there is a penumbra, a set of doubtful cases with respect to which we are incapable of deciding. Furthermore, the extent of this penumbra is not exactly delimited, so that which cases belong to the penumbra is not well defined. In this sense, Russell must have held the existence of higher order vagueness. Paraphrasing him, one can say that we, little by little, arrive at an objective phase where we finally stop applying the name, but again we are ignorant about its precise location: «no one can say precisely when this stage has been reached» (*Ibid.*).

Anew, the implication of (b) is that we shall be in doubt at the moment of applying a particular Collor predicate to an object in the penumbra. Moreover, it is not merely that we lack certainty concerning which situation obtains, "p" or "not p". Russell's reservations about the Principle of Excluded Middle are more radical; for him, the PEM is «not true» (63) when vague language is involved. "p" itself is «neither definitely true nor definitely false» (65). Therefore, penumbral cases do not elicit any definite answer. But the failure of the PEM extends to the whole of logic: this is not valid when applied to the terrestrial life. Logic presupposes an exact language.

Lastly, concerning (c), which can be taken as Russell's opposition to the Discontinuity Thesis, I am inclined to think that it may not be construed as a positive adherence to the *G-F* Correspondence Principle, nor as an endorsement of the major premise of the sorites -(CP) in particular-, due to the indeterminacy that we have just seen in the preceding paragraph. His solution to the paradox appeals to the uncertainty of application of the vague word. If we are doubtful about the range of applicability of the predicate F, then there is at least one a_i about which we are not sure whether it is F or not. As a result, we cannot assert at least one major premise, and consequently, the argument does not go through. Of course, the same outcome is obtained if we cast this approach in terms of truth values: there must be at least one sentence "Fa_i" which is indeterminate, etc.

1a.- Assessment

These are the main points I have wanted to highlight in Russell's early contribution to the debate. Now, I need to add my critical evaluation of his views. Firstly, Russell's connection of fuzzy words with a continuum and gradual processes must be warmly approved, as well as his rejection of the denial of the major premise.

Secondly, we deplore Russell's not being wholly consequent with these three points, which would have led him to concede the major premise of the sorites, especially in the form of the Continuation Principle. Admittedly, the main cause pulling him away from gradualism is his maximalist attitude. If this is wrong, as I tried to show in the Introduction, then all grounds for the imputation of the presumed fallacy of verbalism vanish altogether. There is nothing preventing us to attribute fuzziness to the real world. We agree with Mark Colyvan [2001], who contends that verbalism is a fallacy only if there are reasons to expect that the world is not as language says it is (95, 87).

At last, from the falsehood of the absolute version of the Principle of Excluded Middle, Hp∨¬p, it does not follow that the simple version of the PEM, p∨~p, also fails. The former is completely wrong, but the latter is true to some degree. If the PEM is not totally true, it is false to some degree. Fuzzy situations do weakly falsify the PEM. But, to be partially false does not imply to be completely false. Though it is indeed the case that a fuzzy sentence "p" is neither determinately true nor determinately false, this fact alone does not entail the absence of any truth value being ascribed to "p". Additionally, if we allow the truth of the simple PEM, then there is no obstacle to the effective rule of logic in the material, sensible world. Logic does not govern only the celestial world.

2.- Supervaluationism

2a.- Fine

Kit Fine's [1975] paper is the most famous application of the supervaluation technique to the problem of vagueness and the sorites. His article is an attempt to make the involved philosophical notions very precise by means of logical formalization. Our purpose here is to try to understand the intuitive key pieces of his proposal.

2a.i.- The Precisification of Vagueness

Vagueness has several features. Two of the most important ones -in the opinion of Fine- are that it entails penumbral connections, and higher order vagueness. I will deal with them in a moment, but let me begin with another of its identifying traits. For Fine, vagueness is primarily a sort of deficiency in meaning, implying truth value indeterminacy. The *Principle of Bivalence*, that every sentence is either true or false, fails for vague sentences, which are precisely neither true nor false. Vagueness and truth value gaps are intimately linked, though the former cannot be defined by the latter, because there are other sources of alethic indeterminism, such as reference failure. What is distinctive of vagueness as a factor of indeterminacy is that the gap can be closed by a linguistic decision to make the vague expressions more precise in meaning. Accordingly, to precisify an expression is just to eliminate the possibility of a truth value gap due to vagueness. Fine draws an analogy between a vague meaning and an unfinished picture that carries marginal notes for completion. Indeed, he thinks that the ways in which a term can be made more precise are part of its meaning. In other words, the meaning of an expression is a product of its actual meaning -what helps determine its instances and counter-instances- and its potential meaning, i.e., the possibilities of making it more precise. So, though a vague word is initially underdetermined, there are admissible ways to precisify it. This implies a dynamic conception of language, a process by which the deficiency in meaning is removed. Yet, the elimination of truth value gaps does not generate a change of meaning in the vague expression, because the precisifications must be appropriate, as explained in the next paragraph.

In order to formalize the idea of precisification, Fine introduces the notion of a specification space. A *specification* is an assignment of a truth value to a sentence. A *partial* specification assigns either True, False or Indeterminate, whereas a *complete* specification only assigns the definite truth values, True or False. A *specification space* is a set of specifications ordered by a partial relation, '≥', interpreted as extends. A specification u extends another t, $u \geq t$, if u assigns to a sentence the same definite truth values assigned to it by t. To each specification there corresponds a precisification, and to each precisification, a specification. A *space is appropriate* if the *specifications are appropriate*, or admissible, that is -unofficially-, if the assignments are made in accordance with the intuitively understood meanings of the predicates. Officially, admissibility is a primitive concept. In addition to having a *base specification*, i.e., a precisification of which all others are extensions, a specification space must have the *Completeability Condition*: any specification can be extended to a complete specification: $\forall t \exists u \geq t$ (u is complete). This means that it must be always possible to assign a classical truth value to any vague sentence.

Additional requirements on the truth conditions of sentences are the following three. First, the *Fidelity Condition*: a sentence is true for a complete specification iff it is classically true: $t \vDash p \equiv t \vDash p$ (classically), for t complete, where '$t \vDash p$' means that p is true at t. And similarly for the case in which p is false at *p*. Second, the *Stability Condition*: definite truth values assigned at a certain specification are to be preserved in all its subsequent extensions, that is, if p is true at t, and u extends t, then p is true at u; and similarly if p is false. In symbols: $t \vDash p \wedge (u \geq t) \supset u \vDash p$; $t \Dashv p \wedge (u \geq t) \supset u \Dashv p$, where '$t \Dashv p$' means that p is false at t. And third, the *Resolution Condition*: any indefinite sentence can be resolved in any of the two ways (279); it can be made either true or false. not-$t \vDash p \supset \exists u \geq t\ (u \Dashv p)$; not-$t \Dashv p \supset \exists u \geq t\ (u \vDash p)$. By the way, Fine thinks that this bipolar possibility of resolution constitutes a reason for indeterminism, for «a vague sentence can be made to be either true or false, and therefore the original sentence can be neither» (267).

To further clarify Fine's view of vagueness, it is worthwhile to mention that to assert a vague sentence is to assert its precisifications (282). For example, to assert 'the blob is red' is like asserting the scheme 'the blob is R', where R is a variable ranging over all exact properties substituting for the vague property red.

Let us now pass to the notion of penumbral connections. Fine compares them with a seed from which language grows, in that they provide the logical principles that are to be respected throughout the process. Less figuratively, *penumbral connections* refer to the fact that vague sentences stand in logical relations. For instance, suppose that a blob is a borderline case of pink and of red, and let 'p' mean 'the blob is pink', and 'r', 'the blob is red'. Then, though both 'p' and 'r' are indefinite, the disjunction 'p or r' is true because the predicates 'pink' and 'red' are complementary, while the conjunction 'p and r' is false, since those predicates are contrary. In effect, these two compound sentences, the disjunction and the conjunction, are equivalent to saying that the blob is pink or not pink, and that it is pink and not pink, respectively. While the former is an instance of the Principle of Excluded Middle, the latter is a contradiction. But the blob cannot be made a clear case of both, pink and red (271). Fine explicitly demands that the extensions of the predicates 'pink' and 'red' be such that, once precisified, they do not overlap (277). Hence, the Principle of Non-Contradiction and the PEM must remain valid, even for vague sentences, contrary to what happens in some many-valued and fuzzy logics. Consequently, if it is possible that 'p or r' is true, and 'p and r' is false despite the fact that both p and r are indefinite, then the functors will not be truth-functional: the truth value of a disjunction or conjunction will not depend on the values of their immediate sub-sentences. The connectives in general will not have classical truth conditions. By renouncing truth-functionality, it is feasible to maintain an indeterministic view of vagueness and at the same time to keep classical logic.

We have seen so far that vagueness is a species of meaning and alethic indeterminacy, and that it involves penumbral connections. The third basic attribute of vagueness, according to Fine, is higher order vagueness, that the vague is vague. That is, the boundaries of the borderline cases of a vague term are blurry. Whether a vague sentence has an indeterminate alethic status may itself be indeterminate. To express higher order vagueness, he defines an *indeterminacy operator*, by means of the determinacy operator:

$$I\text{p} =_{\text{df}} \neg\Delta\text{p} \wedge \neg\Delta\neg\text{p}.$$

That it is indeterminate whether p is indeterminate, IIp, is *second order vagueness*. If a sentence can be indeterminate, then it can also be indeterminate that it is indeterminate. In this instance, «the higher order consists in the correct application of I to a statement of indefiniteness» (288). Fine claims that there is no halt to the number of times the indeterminacy operator may be reiterated. If higher order could end at some stage, that would mean that vagueness could be eliminated at some level.

In conclusion, Fine argues that his position is superior to others because it is the only one that accommodates all penumbral connections and the reasonable conditions, such as fidelity, stability, completeability and resolution.

2a.ii.- Super-Truth and Validity

In order to analyse the sorites, the notions of truth and valid argument need to be spelled out. The definition of truth is based on the notion of a complete and admissible specification, and therefore, truth is relative to a space, and depends on ways of making the meaning more precise.

> A sentence is true simpliciter if and only if it is true at ... all complete and admissible specifications (272-3).

In this respect, vague and ambiguous sentences have similar truth conditions. In fact, an ambiguous sentence is true if it is true in all its disambiguations. In the same manner, a vague «sentence is true if it is true for all ways of making it completely precise» (278). No matter where one draws the line to the extension of a predicate, provided that the boundaries respect the intuitive original assignments, a sentence is *true* iff it results true in every manner of drawing the boundaries. This is what is meant by saying that truth is *super-truth*. And exactly the same conditions required for "p" to be true are demanded for "definitely true" (293). Therefore, that "p" is true is equivalent to that "p" is definitely true (295). And analogously, a sentence is false if it is false in all complete and admissible specifications.

As for the notion of *valid argument*, Fine provides the following definition. "q" is a *consequence* of "p" if for any specification space, "q" is true whenever "p" is true. That is, a valid argument preserves super-truth.

2a.iii.- Why the Falsity of the Major Premise Does Not Entail Sharp Boundaries?

Fine believes that [the conditional version of] the sorites reasoning is valid but the major premise is false (285). In fact, let us consider the premise "if a man with n hairs on his head is bald, then a man with $n+1$ hairs on his head is bald". For a lot of cases, whether a man with n hairs on his scalp is bald or not will be indeterminate unless one precisifies the predicate. Hence, the evaluation of both, the antecedent and consequent of the conditional is relative to all precisifications, in accordance with the definition of truth as super-truth. Now, every precisification draws a line at some point n, which varies from one precisification to another, in such a way that a man with n hairs is bald, but a man with $n+1$ hairs is not bald. Thus, it is true on each precisification that there is a hair number n, which makes the difference between the bald and not bald, and therefore, it is true simpliciter that there is a splitting hair. Consequently, the major premise is false on each precisification, and then, it is false simpliciter. However, Fine contends, that from this it does not follow that 'bald' is precise for the splitting hair n is not the same for all precisifications, but differs from one precisification to another. So, the paradox is dismantled, and there is no need of a special kind of logic of vagueness other than the classical one. What has been altered is only the bivalent semantics.

Let me finish this overview with a quote in which Fine states the reasons for favouring a classical solution.

> The first is that it is a consequence of a truth-definition for which there is independent evidence. The second is that it can account for wayward intuitions in an illuminating manner. And the last is that it is simple and non-arbitrary (287).

These are the main theses upheld by Fine.

2b.- Keefe's defence of Supervaluationism
Rosanna Keefe [2000], chapters 7 and 8, constitute a detailed defence of supervaluationism. We will examine in the present section the major criticisms launched against the theory together with her replies to them.

OBJECTION 1.- In as long as supervaluationism renounces the truth-functionality of the functors, like conjunction and disjunction, it distorts or misinterprets the meaning of English words 'and', 'or', etc., and therefore it changes the common understanding of the universal and existential quantifications.

REPLY.- One should bear in mind the motivation behind the non truth-functionality. It was a required move within a theory that tries to accommodate, on the one hand, «the fact» that vague sentences are neither true nor false (219), and, on the other hand, the desideratum to preserve classical logic unaffected. Thus, one can have penumbral connections without bivalence. So, in order to have both, indeterminism and the principle of excluded middle, truth-functionality would have to go. Furthermore, the loss of truth-functionality and other semantic anomalies are collateral effects that should be accepted as part of an overall theory that successfully deals with the phenomenon of vagueness. When one takes into account the whole performance of the theory, its effectiveness in dismantling the sorites paradox while at the same time respecting borderline cases and lack of sharp boundaries, that make the core intuitions of vagueness, then, when looked upon against this package of benefits, the forfeiture of truth-functionality will be judged as nothing more than a small cost to be incurred.

OBJECTION 2.- By falsifying the main premise of the sorites, namely, $\forall i\ \neg(Fa_i \wedge \neg Fa_{i+1})$, supervaluationism endorses its negation, the truth of $\exists i\ (Fa_i \wedge \neg Fa_{i+1})$. And this seems to assert the existence of a sharp cut-off point, in flat opposition to what vagueness means.

REPLY.- Indeed, the commitment to the falsity of the major premise, and the corresponding truth of its negation are the least appealing aspects of the theory (183). But several things can be said in their support. First, the sorites paradox teaches us that there is a set of separately plausible assumptions, but which jointly lead to an absurd conclusion. To avoid this predicament, any theorist of whichever persuasion is forced to relinquish at least one of the initially sound presuppositions, so that there is no theory without disadvantages. All approaches will have some counter-intuitive results. Second, due to the failure of truth-functionality, there are two distinctions that need to be drawn: the falsity of a universal quantification is one thing, the falsity of its instances is another; and similarly, the truth of an existential quantification is not the same thing as the truth of any of its instances. Thus, though it is true simpliciter that $\exists i\ (Fa_i \wedge \neg Fa_{i+1})$, this does not entail that, for some particular i, it is true simpliciter that $Fa_i \wedge \neg Fa_{i+1}$, strange as it may sound. This situation is parallel with the one in which a disjunction is true without any disjunct being true. Hence, the simple falsity of the major premise is not too bad after all, thanks to the fact that none of its instances is false simpliciter.

However, if the major premise of the sorites is false, there remains the task of explaining why is it that it looks like a perfectly admissible proposition, at least prima facie, or that we mistakenly believe that it is true. And again, the distinctions just made provide their services here. The truth of the universally quantified premise is merely apparent. Because $\neg(Fa_i \wedge \neg Fa_{i+1})$ is not false of any instance, people erroneously assume that its universal generalization ought not to be false. And in the same manner, from the fact that there is no true instance of $Fa_i \wedge \neg Fa_{i+1}$, people wrongly conclude that its existential generalization is not true.

OBJECTION 3.- Supervaluationism is not fit to represent vagueness because it cannot make true the following two propositions, (a) 'predicate '*F*' lacks sharp boundaries', and (b) "*F*' has borderline cases', for, on each precisification, '*F*' does have sharp boundaries, and there is no borderline cases. Then, both sentences are false on each precisification, and, consequently, they are false simpliciter. Even the statement that (c) 'predicate '*F*' is vague' comes out simply false, for '*F*' is precise on each precisification. So, supervaluationism sacrifices vagueness. There are serious doubts concerning its ability to accommodate lack of sharp boundaries.

The situation is still worse since the theory prohibits its own enunciation; that is, the description of the system goes against its own principles. In fact, according to supervaluationism, vagueness manifests itself in the multiplicity of admissible precisifications (207). However, on each precisification, there is only one way to make the predicate '*F*' precise, so that the affirmation (d) 'there is a multiplicity of ways of precisifying '*F*'' turns out false, according to the lights of the theory itself, and hence, a unique extension should be assigned to '*F*'. Thus, supervaluationism suffers from problems of ineffability: if the theory is true, it cannot be enunciated. But a conception that does not allow its own enunciation has very severe limitations.

REPLY.- The four statements above, (a)-(d) (that predicate '*F*', lacking sharp boundaries and having borderline cases, is vague but can be made precise in several ways), are not ordinary, object language propositions, but metalinguistic statements, which should be formulated with the help of the Δ operator, or the truth predicate. For example, the affirmation that '*F*' *lacks sharp boundaries* should be expressed as:

(a') it is not the case that, for some i, 'Fx_i' is true but 'Fx_{i+1}' is false,

or as:

(a'') it is not the case that, for some i, $\Delta Fx_i \wedge \Delta\neg Fx_{i+1}$.

Again, the statement that '*F*' *has borderline cases* should be interpreted as:

(b') for some x, neither 'Fx' nor '$\neg Fx$' are true.

Now, given that (a') and (b') are not object language assertions, they cannot be evaluated at precisification s just by the behaviour of '*F*' at s. Rather, the alethic status of (a') and (b') on precisification s depends on the structure of the whole specification space. Their evaluation requires our taking into account what happens in all precisifications. And, there, they result true.

Additionally, there are at least three considerations in favour of supervaluationism being in a position to accommodate lack of sharp boundaries. First, vagueness is understood in terms of a multiplicity of admissible precisifications. But the range of these multiple precisifications is not clearly delimited. Second, the notion of an admissible precisification of the expression '*F*' is nothing but an acceptable way of making '*F*' precise. And one would naturally expect that what counts as acceptable is vague. Thus, what is an admissible precisification is also vague, and hence it can have borderline cases. Third, and most importantly, there is no way to definitely determine the truth status of all members in a soritical series. With respect to some of them, one cannot assign a definite truth value, either true or false, nor definitely assign lack of truth value. It is not always possible to definitely classify the elements in the series, even if one avails oneself of the category 'neither true nor false'. In other words, what is definite and what is indefinite is itself indefinite. There is higher order vagueness.

OBJECTION 4.- There is something particularly inadequate in the supervaluationist's use of precisifications to represent vagueness, for, if a vague sentence 'p' is indeterminate, why should one expect to gain some illuminating insight by appealing to precisifications, in which 'p' is either true or false? If natural language is vague, it seems that investigating what would happen if that language were made precise is irrelevant to the enterprise of understanding vagueness (Cfr. Burns 1991: 69).

REPLY.- It is true that predicates of natural language are vague, but, once they are precisified, they have become precise on each individual precisification. However, this is no objection to supervaluationism for it only claims that the meaning of vague predicates is captured by quantifying over all precisifications. Thus, the objection misconceives the role that precisifications play in the theory. The set of precisifications gives you certain freedom to choose where to place the dividing line.

OBJECTION 5.- Supervaluationism must abandon the *Tarski schema*, (T): 'p' is true iff p. One reason for this is that, (T) together with the principle of excluded middle, which supervaluationists admit, entail the principle of bivalence, which they reject. Another reason is the following. Suppose that 'p' is true at point *s*, but not at all points. Then 'p' is not true simpliciter, and thus, "p' is true' is not true at all points. Hence, at *s*, the right to left direction of (T), namely: that p entails that 'p' is true, fails, for it has a true antecedent but a false consequent, and so (T) itself cannot be true.

REPLY.- Yes, there are instances of (T) that are not true. In fact, once it is admitted that truth value gaps are possible, the evaluation of "p' is true' is not straightforward for the case in which 'p' is indeterminate. There are conflicting judgements here: on the one hand, "p' is true' might be considered also indeterminate, to mirror the situation of 'p'; on the other hand, it might be assessed as false, for, if 'p' is neither true nor false, then 'p' is not true. So, when the right member of (T) is indeterminate, it is possible that the left member is false, and therefore, the right and left members of (T) do not necessarily have the same truth value. Nonetheless, this result is not so grave. For one thing, (T) is never false. For another, though the biconditional version of (T) fails, there is another version which does not, to wit: from 'p' is true, one can infer p, and vice versa. Keefe symbolizes this as

(T*) 'p' is true $\vdash$ p; and p $\vdash$ 'p' is true.

She claims that (T*) is faithful enough to our intuitions about truth.

2c.- Assessing Supervaluationism

Once that we have seen the main elements of supervaluationism, and had an idea of the on-going discussion, let us pause a moment to critically ponder its value, both as a conception of vagueness and as a solution to the sorites paradox.

To begin with, it must be admitted that its dissatisfaction with strong bivalence is justified. Vague sentences do not fit into the absolutist, binary classification of the received logic. We agree that the classical semantics must be revised to make room for a third category that is neither definitely true nor definitely false, but in such a way that the revision does not sacrifice any classical tautology, theorem or rule of inference. Fuzziness demands an extension of Classical Logic so that a new kind of alethic status is to be added to the standard ones.

Having said that, we need to indicate our reasons against the particular way supervaluationism takes to reach the goal of accommodating vagueness without weakening CL. First, I express my dissent from the supervaluationist attribution of indeterminate status to vague sentences. Let us suppose that Frank is a borderline case of baldness. This implies that he is not a clear case of a bald person, nor is he a clear case of a hairy person. Thus, he is neither definitely bald, nor definitely non bald. So far, we agree, provided that we replace the over affirmation functor 'H' for the 'definitely' operator. We are using the definitely operator to adequately express a fuzzy situation. Now, some supervaluationists and other indeterminists contend that a borderline case can be precisified either way. That is, *a* may legitimately be put either in the extension of the predicate '*F*', or in its anti-extension. There is nothing in the world, in our use of language, or in the meaning of the predicate that prohibits either alternative. There is leeway or liberty here, permitting the resolution of the indefiniteness in both directions (Sainsbury 1988: 31; Beall and Van Fraassen: 138). In some precisification, Frank will be bald, but not in another. Yet, both precisifications may be admissible, not offending any principle.

Let me quote some relevant passages in the literature supporting this construal. We saw already that Kit Fine (1975: 267) maintained that the possibility of a sentence to be made either true or false entailed its being indeterminate. Moreover, the Resolution Condition states that any indefinite sentence can be resolved in any of the two classical ways (279). For Manfred Pinkal (1995: 53), a necessary and sufficient condition for a sentence to be indefinite in truth value is that it can be precisified alternatively as True or False. He believes that True or False cannot be definitely assigned to an indefinite sentence because both values are plausible (*Ibid.* 18, 139). A similar conviction is held by Dorr (111-12, note 25): a sentence p is indeterminate not because p and ~p are forbidden but because both p and ~p are permitted. Again, Crispin Wright holds that there is a coherent indeterminist characterization of borderline cases. Indeterminacy is conceived as a circumstance in which things are left open, or unsettled. Competent judges may permissibly differ about a borderline case. For example, if a patch is in the red-orange border zone, that borderline case is consistent with being red and with being orange, because it has not been determined (1994: 138-9). Hence, it is consistent to suppose each of the parties in a dispute to be right (*Ibid*. 146). Analogously, Linda Burns [1991] maintains that in borderline cases, the application of the predicate '*F*' to the object *a* is undetermined (3, 49, 54) in the sense that all the facts are insufficient to establish whether *a* is *F*; and that the rules of use are loose, allowing some scope for individual judgement in difficult cases, for «confronted with a borderline case, it does seem equally correct to go either way» (135). On the other hand, Michael Tye, describing a particular version of supervaluationism, gives the following example. Alfred is 66 years old. The meaning of the predicate 'old' depends on how it is precisified. It may mean to have at least 65 years, or, alternatively, 68 years, to consider no more than just two possibilities. According to the first precisification, Alfred is old, but he is not old according to the second precisification. Now, if both ways of making the predicate precise are «equally acceptable», then, it is indeterminate whether Alfred is really old (1994b: 13). And, finally, Roop Rekha Verma (67) says of a borderline case that both of the supposedly contrary classifications appear equally proper and no objective basis for preferring the one over the other is found; and thus, the case remains indeterminate.

So, some supervaluationists and indeterminists sustain that a vague sentence, "p", is indeterminate for it is permissible to ascribe to it either of the two classical truth values. The initial status of "p" is unsettled; but the lack of truth value is temporary, or provisional, being always possible to be removed by a precisification respecting certain constrains. Thus, whether a vague sentence is true or not is conditional on where you draw the line (Shapiro, Ch. 1, Sect. 3). The fact that a vague sentence is made true or false becomes relative to a precisification.

However, I would like to dispute the claim put forward by Burns and Verma that both ways of resolving the indefiniteness appear equally correct. A more general claim is found in Bittner and Smith (2001: § 2), who affirm that all admissible precisifications are equally good. The underlying thought behind these "egalitarian" theses seems to be that there are reasons in favour of placing the borderline case *a* in the extension of the predicate *F*, reasons that are as appealing as those suggesting the placement of *a* in the anti-extension of *F*. Because both tendencies are *isocratic*, i.e., of equal power, each would eliminate or neutralize the other, resulting in an indeterminate predication. But, if this is the motive behind the indeterminacy thesis, it is not convincing for a friend of paraconsistency. Even if *a* is the middle point of a soritical series, whose location constitutes a ground for its being classified as an *F* as much as a non *F*, its situation is not one of indefiniteness at all. Being halfway between *F* and not *F*, *a* partially partakes of both extremes. *a* is contradictory, it is 50% *F*, and 50% not *F*. So, one can conclude that *a* is neither *F* nor not *F* only if one presupposes that its condition of being *F* and not *F* is absurd, an assumption that has not been justified. The attitude of these supervaluationists and indeterminists resembles that of some ancient skeptics, who renounced to the aspiration of truth in light of the fact that opposing schools were equally persuasive, or because the object appeared to them as *F* as not *F* (See Chapter 6, sections 6e and 6f, for more on the contradictoriality of borderline cases).

And with respect to the other borderline cases, those that do not occupy a middle point in the soritical series, their situation is much less perplexing, since they are closer to one of the poles than to the other. Their position makes it easier to place them in either the extension or anti-extension of *F*, depending on their distance to one of the extremes. So, neither usual supervaluationists nor indeterminists can account for the differentiation between borderline cases (Sanford 1976: 210; Edgington 2001: 377; Cfr. Clark 1987: 178).

Therefore, there is a paraconsistent rationale to oppose the supervaluationist's claim that borderline cases are indeterminate. We need to assert that intermediate cases are really contradictory.

A second basis to disbelieve supervaluationism is that it violates the *similarity principle* (that in a soritical series, for any pair of elements that are only marginally different with respect to the features relevant to the application of *F*, either both are *F* or neither is), since there must be at least one couple of adjacent members in the series that do not receive a similar treatment (Cfr. Smith 2001: 50). For example, in a sequence going from *F* to not *F*, there must come a pair of neighbouring members, a_i and a_{i+1}, such that only the former is *F* but not the latter, for a_{i+1} may be indeterminate. And it is clear that to say that a_i is *F* while at the same time saying that it is indeterminate whether a_{i+1} is *F* does not respect similarity. Even if one abstains from judging whether a_{i+1} is *F* or indeterminate, not taking a stand over the status of a_{i+1}, one does not avoid breaking the similarity between the two contiguous elements. Not making a pronouncement on what a_{i+1} is does constitute a different response from the treatment accorded to a_i. The same conclusion is reached if we consider the precisifications of the predicate.

Third, pace Keefe, supervaluationists have not managed to appease the feeling of discomfort caused by their affirmation that a vague expression has no sharp boundary, since the mere fact of multiplying the sharp boundaries, even if one imposes strict requirements for their admissibility, does not erase the fact that the expression continues to be precise. If a single sharp boundary is already something alien to the essence of vagueness, several such boundaries do not ease their inaptness, but reiterate their unsuitability. Expanding the range of options of where to draw the line does not alter the plain fact that, after the precisifications, all previous borderline cases have been annihilated. Thus, I take it that there is something wrong in the project of replacing vagueness by precision.

And, finally, the loss of truth-functionality of the functors, together with its alteration of the meanings of the quantifiers, plus the surrendering of the Tarski schema, put supervaluationism in a further disadvantageous position.

3.- Michael Tye's Indeterminism

Indeterminist positions are currently very popular. One of its compelling defences has been made by Michael Tye, whose case we offer in the present section. One particularity of his thought, in contrast to supervaluationism, is that he believes that there is vagueness not only in language or in our thought but also in reality itself, more exactly, concrete objects, properties and relations can be vague.

The underlying logic is Kleene's three-valued system. The intermediate truth value, symbolized 'I', is not really a third truth value but signals an absence of truth value: it represents that the sentence to which it is assigned is neither true nor false. Notwithstanding, loosely speaking, we will continue talking of 'I' as if it were a truth value. The presence of this indefinite truth value has the consequence that no classical tautology is preserved in the system. In order to somehow recuperate all classical truths, the notion of a *quasi-tautology* is introduced. This is any formula that never receives a value false. Thus, classical tautologies become quasi-tautologies in Tye's theory.

There are two peculiarities of Kleene's system that are worth noting. First, the negation of "p" is indefinite when "p" is indefinite. This valuation for the case in which "p" gets an intermediate value makes the Kleene negation a sort of weak negation. We symbolize it as "∼p". And second, the conditional is defined as "∼p∨q". We will use "p⊃q" for this conditional, as exemplified in the next matrix for a pentavalent logic.

⊃	1	¾	½	¼	0
1	1	¾	½	¼	0
¾	1	¾	½	¼	¼
½	1	¾	½	½	½
¼	1	¾	¾	¾	¾
0	1	1	1	1	1

Notice further that, when "p" is indefinite, the self-entailment formula, "p⊃p", and a contradiction, "p∧∼p" both result indefinite, instead of true and false, respectively. Tye contends that the mistake would be to, respectively, assign false and true to these formulas. In as long as 'p' is a borderline case, there are good reasons to oppose the traditional assignment of true to 'p⊃p' and false to 'p∧∼p' (1994a: 282-83).

5a.- There Is No Determinate Fact of the Matter about whether *a* is *F*

Let us continue now with his conception of linguistic vagueness. Singular and general terms can be vague. An expression is vague if it has borderline cases of application. But this condition is not sufficient due to the phenomenon of higher order vagueness, as we will see later. The ordinary, pre-philosophical notion of a borderline *F* implies that there is an object *a* such that it is neither definitely *F* nor definitely not *F* (1994b: 18). Philosophically, this is captured by the sentential operator "there is no determinate fact of the matter about whether" *a* is *F*, and this operator is not reduced to 'it is indefinite that', nor to 'it is neither true nor false that'. A case in point is the definite description 'the friend of Amy'. Suppose that Amy has a love-hate relation with respect to Jane, and that there is no other person that is clearly a friend of Amy. Then 'the friend of Amy' vaguely designates Jane, in that it is neither true nor false that it designates Jane. The law of excluded middle is not true. It is not true that the definite description refers to Jane in virtue of Amy's bearing the hate relation to Jane; but neither is it false that it refers to Jane thanks to their being in the love relation. So, Jane is a borderline case of application of 'the friend of Amy', and so this definite description is vague (1994b: 2).

Furthermore, as was already anticipated, vagueness is not merely a characteristic of our representations, but an intrinsic aspect of the world. Metaphysical vagueness can be found in concrete ordinary objects, properties and relations. Vague objects are those that have or could have borderline spatio-temporal parts. For example, mountains, clouds, deserts, islands, etc. all have fuzzy boundaries: there is no exact and precise single boundary that could be claimed to be *the* boundary of the object in question. A popular illustration of this is the Everest mountain. There are some rocks that certainly are part of the mountain; other rocks are certainly not part of Everest; and there are still others with respect to which there is no determinate fact of the matter about whether they are part of Everest or not. This is vagueness as a feature of objects. On the other hand, a property *F* is vague if it has or could have borderline instances, i.e., when there is an object that it is indeterminate whether '*F*' applies to. Thus, it is licit to infer the vagueness of a property from the vagueness of the predicate designating it. But again, a proviso is needed to the effect that higher order vagueness should be taken into account. See at the end of the next sub-section for more on the required complement.

Tye insists that fuzziness is not epistemic, involving simple uncertainty. It is not merely a question of our lacking relevant information permitting us to correctly classify the object as *F* or not *F*. The operator 'definitely', that enters in the ordinary account of a borderline case, does not mean 'knowably' or 'known'. The sense of 'definitely p' is explained as a logical consequence of 'it is a fact that p'. Nor is vagueness a semantic phenomenon, pertaining to the meaning of an expression; rather, it has ontic connotations.

We have thus seen that vagueness has been characterized as there being no determinate fact of the matter about whether a certain object has a property, and as lack of sharp boundaries. Because of this second trait, Tye says that vagueness is *robust* or *resilient*: it is never the case that vagueness implies sharp boundaries. Tye (1994b: 18) calls our attention to the fact that Timothy Williamson has denied the existence of robust vagueness.

5b.- Indefinite Premises and Higher Order Vagueness

Before expounding Tye's own view of the paradox, it is important to realize what he wants to avoid, since this repudiation constitutes part of the motivation for his proposal. In fact, he refuses to admit that the sorites should be regarded as a reductio of the major premise, to wit, for all a_i, if a_i is *F*, then a_{i+1} is also *F*. It is often voiced that, if this premise were false, then its negation would be true, namely, there would be an a_i such that a_i is *F*, but a_{i+1} is not *F*. This in turn would entail the existence of a sharp boundary, and hence, *F* would not be vague, but precise. Hence, '*F*' could not refer to a vague property. There would not be vagueness neither in reality nor in language. But all this is hard to accept. On the contrary, Tye holds that we know that there is vagueness. Therefore, the major premise cannot be false. Indeed, Tye says that the meaning of the major premise guarantees its non falsity (1994b: 15). However, on the other hand, the major premise cannot be true either, due to the absurdity of the conclusion: «Since the conclusion of the argument ... is false, not all of the conditionals are true» (*Ibid*.). So, what can legitimately be deduced from the paradox is that at least one of the premises is not true. More precisely, at least one of the conditionals will be indeterminate in truth value. Thus, the indefiniteness of the major premise is a result of the pressure to eschew both the paradox as well as a sharp dividing line.

According to Tye, the universally quantified conditional major premise is equivalent to the denial of a cut off point in the soritical series: there is no a_i such that a_i is *F*, but a_{i+1} is not *F*. If the universal quantification is not true, then so is the negative existential. But take notice that the non truth of $\sim\exists a_i\ (Fa_i \wedge \sim Fa_{i+1})$ does not entail the truth of its negation, namely, $\exists a_i\ (Fa_i \wedge \sim Fa_{i+1})$, for, actually, both are indefinite. So, the question: 'is there a cut off point?' does not have any definite answer.

Another rationale for considering the major premise indefinite is that there is an assignment of truth values to the conditional in such a way that both its antecedent and its consequent get an indefinite status. The reason for this assignment is that, if a_i is a borderline case of a bald person, then the predication 'Fa_i' will be indeterminate. And, if a_{i+1} has one more hair than a_i, then the extra hair cannot make the person a_{i+1} cease to be a borderline case, and 'Fa_{i+1}' will be indeterminate too. Consequently, the entailment '$Fa_i \supset Fa_{i+1}$' gets also the same indefinite truth value, in accordance with the conditional truth table.

Hence, the sorites is *not sound*: it is valid but it does not have true premises. By the way, the concept of *valid argument* upheld by Tye is a little bit more complicated than the classical one: it cannot be true that the truth value of the conclusion is other than true when the value of the conjunction of the premises is true (1994a: 283, n. 3). And what is most important, the paradox is avoided without a commitment to a sharp dividing line.

It might be objected against this indeterminist conception that, if there is at least one conditional that is not true, then there must be a couple of adjacent conditionals such that the first is true but not the second. That is, there would be a pair of contiguous members in the soritical series such that 'Fa_i' is true but 'Fa_{i+1}' is not true, being rather indefinite or false. Yet, this will clearly introduce a sharp division in the sequence, contrary to vagueness.

Tye's reply to this charge is to appeal to higher order vagueness. According to him, the objection employs an unsupported assumption, namely, that every sentence in the soritical sequence is either true, false, or indefinite. We may dub this presupposition '*the principle of excluded fourth*' (PEF). Tye's claim is that this tripartite disjunction is itself indefinite. He says that the truth value predicates are vaguely vague. It can be indeterminate whether a sentence is true, false, or indefinite. The PEF is not true, because that would presuppose that the division between the three truth values is sharp, every sentence fitting neatly into one of the three categories. But the PEF is not false either, for that would require the addition of more truth values. So, it may be indeterminate what alethic status 'Fa_{i+1}' has, even if one avails oneself of the category of 'neither true nor false'. It may be indeterminate whether the sentence is indefinite. This line of defence gets additional evidence given that people do not agree on where the lines should be drawn between those members of the soritical series that are *F* and those that are not. Even the same person may draw the limit at different places on different occasions. So, there is no definite dividing line separating the *F* cases from those that are not. Again, there is no determinate fact of the matter about where the transition occurs, because that is indeterminate. Therefore, there is no first conditional whose truth status is not true.

5c.- Criticism

Tye's indeterminist position is well argued for. He holds some theses that we adhere to, such as that there are no sharp boundaries, that fuzziness is not merely a matter of our representations, but that the world is fuzzy (though we understand this latter assertion in a different sense), and that there are good reasons to deny that "$p \wedge \sim p$" is False.

Notwithstanding, we are bound to disagree with him on a number of other points. First, concerning the nature of vagueness, Tye maintains that it is responsible for truth value gaps. Though we also think that a vague sentence, "p", is neither definitely true nor definitely false, that does not entail that "p" entirely lacks a truth value, unless -it seems to me- one presupposes *strong bivalence*: that there are only two truth values, that are jointly exhaustive and mutually exclusive. It is true that a vague sentence does not receive any of the classical valuations, but, from this fact alone, it does not immediately follow that there are no other truth values that could be assigned to the vague "p". Really, I am unable to see how else he could conclude that the vague sentence "p" is indeterminate if not by a sort of elimination of cases, like this: "p" is neither true nor false; but there is no other truth value that can be assigned to a sentence; therefore, "p" lacks any truth value. Perhaps, the only sense in which the indeterminist makes the supposition of bivalence is that she takes it as a hypothesis for a reductio ad absurdum. Needless to say, I am not charging the indeterminist with incoherence, but simply paraphrasing an objection raised by Sorensen (1992: 181), who claims that, by limiting the set of possibilities considered, one increases the appearance of indeterminacy. Consistent with his indeterminism, Tye declares that he neither asserts nor denies the principle of bivalence (1994b: 17). However, I think he implicitly uses this principle to conclude that there are truth value gaps due to vagueness.

Second, if we think about the reasons why a sentence is indeterminate, it seems that Tye confuses having reasons in favour of both "p" and "not p" with not having reasons for any of the contradictories. For example, when he gives an instance of a vague definite description, he contends:

> Suppose Amy has a love-hate relationship with Jane, without Amy having any other friends. Then, it is indeterminate whether 'the friend of Amy' designates Jane (1994b: 2).

It would appear that it is not a complicated inference to conclude that a situation is indeterminate from the supposition that it is contradictory. The reader is left to figure it out how the deduction proceeds in detail, but Tye does not provide any clue. Admittedly, it is easy to see why it is not true that Jane is the friend of Amy: because Amy bears the hate relation with respect to Jane; and the reason why it is not false that Jane is the friend of Amy is that they are in a love relation. Thus, it is neither true nor false that Jane is the friend of Amy. This is how I have reconstructed the implicit proof. But the inference is not so straightforward, for the situation could also be judged contradictorially. Thus, Jane is the friend of Amy since they bear the love relation, and at the same time Jane is not the friend of Amy because Amy has the hate relation with respect to Jane. Hence, Jane is and is not the friend of Amy. This conclusion just makes it explicit the tacit contradiction involved in the supposition. When we are invited to imagine that Amy has a love-hate relation with Jane, we suppose that Amy loves Jane while simultaneously hating Jane. Thus, I conclude with Batens (2000: 53, n.) that a situation in which one has reasons to believe "p", and reasons to believe "not p" is different from a situation in which one has no reasons to believe either "p" or "not p".

Third, it seems that Tye has not managed to avoid a sharp boundary in the soritical series. If a_i is F, what is the alethic status of 'Fa_{i+1}'? Initially, we have three possibilities: true, false, and indefinite. But Tye has urged that it is indeterminate whether the vague sentence in question is true, false or indefinite. However, it seems that the similarity between Fa_i and Fa_{i+1} is lost given that the two adjacent members do not receive the same treatment. The similarity is broken as long as 'Fa_i' is true, while 'Fa_{i+1}' is something else other than true, even if it is assigned a second order indefinite status.

Fourth, it seems that, when one is confronted with a borderline case, and asked whether an object is *F* or not *F*, there are better options than to remain silent, without putting forward any opinion. One may hedge, and answer, for example, 'it is sort of *F*', 'it is around ¾ full', etc. (Dorr: 86-7; Simons 1992: 169).

CHAPTER 4

THE MANY-VALUED, FUZZY VIEW

In this chapter we will take a look at the many-valued, fuzzy approach of vagueness and the sorites paradox. We begin with Rayme Engel's convincing defence of the existence of gradual properties. Then we describe the pioneer work of Goguen and Lakoff, who appeal to fuzzy logic, though both sustain maximalist and indeterminist tenets! Next, Machina's theory is presented, followed by Smith's view. Each of these authors have powerfully contributed ingredients to the development of a gradualist conception of fuzziness. We are indebted to them all.

At the end of this chapter, we will discuss some famous objections to the notion of a degree of truth.

1.- Engel

Rayme Engel [1989] presents a vigorous defence of degrees of possessing a property. It is the purpose of the current section to introduce the main ideas of his article. I will first expound his own view, and then his critical remarks about the non gradual outlook.

1a.- There Are Gradual Properties

Engel thinks that the existence of at least two distinct positive degrees of possessing the property of being wise is established by showing that *a* is wiser than *b*, and that *b* is wiser than *c*. In a situation like this, we are capable of distinguishing the extent in which *a* and *b* are wise. Though both, *a* and *b*, have the property, *a* has it to a greater extent. Again, some things are more flammable than others. We may wish that p, and wish that q, though we do not wish q as much as p. Similarly, the existence of degrees is suggested by the fact that there is an increase or decrease of a property, or whenever we can sensibly ask how much or to what extent a property is instantiated.

If this is so, then linguistic competence in using the gradual expression '*F*' requires from us not only to recognize whether a given object is *F* or not, but also the ability to discriminate the extent to which the property *F* may be possessed. It is not that properties are simply present or absent, but rather their presence or absence can be realized in a variety of intensities. It is not a question of all or nothing.

Other examples of properties coming in degrees are the following: honesty, precision, immunity. Many of the philosophical concepts are amenable to be possessed to different extents, such as: certainty, refutability, objectivity, meaningfulness, goodness, virtue, injustice, consciousness, freedom, responsibility, personhood, etc. Quine even has gone further in his demand that analyticity be considered gradual.

The fact that a property can be possessed to different extents does not imply that we are immediately aware of its various degrees. So, we may perceive a stuff as flammable even if we do not think about its particular intensity to be flammable. That being *F* is a matter of degree is not always conspicuous in that having an object that is *F* before our mind does not always invite a question of how much *F* it is. It is easier to identify a matter as one being subject to a variation by degrees when there has been for a long time the means to measure it. Whenever it is the case that one can identify the precise degrees to which some objects exemplify a property *F*, the graduality of *F* is more apparent. Matters of degree thrive with increased precision. When we have at our disposal advanced means to measure the possession of a property *F*, the more evident it is that *F* can be exemplified to different extents.

The property of being gradual is itself a matter of degree. Some properties are extensively gradual while others are minimally so. The scope of gradual properties is so vast that Engel postulates its almost universality. Indeed, he lays down the following thesis, which resembles Anaxagoras' thought.

(TD) Virtually everything is a matter that admits of differences in degree.

The degrees are practically so ubiquitous that apparent discontinuities are not discontinuous enough. Even those changes that seem to be abrupt are gradual: they must take place through intermediate stages. Irrespective of where the dividing line is drawn between the poor and the rich, the degrees of both are not obliterated.

Closely related with the (TD) is Leibniz' *principle of continuity*[14], that properties must admit of varying degrees of being possessed, for otherwise, there would not be a continuous change from one to the other. In the absence of degrees of being rigid, there could not be gaining nor losing rigidity. If stiffness did not lend itself to come in degrees, one could not understand how something may stiffen. It is said that Leibniz used the principle of continuity to discover the existence of zoophytes, i.e., organisms that are to some extent both plants and animals.

Unfortunately and curiously, things that are a matter of degree are treated as if they were a matter of kind. But even biological kinds must be gradual. For instance, the species of cats must have evolved gradually from its chain of ancestors till the pre-cats and first cats. That there are degrees of being a dog is evidenced by the continuity in the process that begins with the conception and ends with the birth of the baby dog.

Finally, I allude to the inverse relation of the degrees of opposite properties. *x* is larger than *y* iff *y* is smaller than *x*.

1b.- Against Dichotomies

So far, we have seen reasons for believing that most of everyday properties admit of degrees. Now, we add a few critical comments on the opposite point of view. If *F* is a matter of degree, then to define it by giving necessary and sufficient conditions, omitting questions with respect to the extent to which some object may be *F*, will result in an incomplete or partial definition. To mention the extent to which a property can be exemplified is indispensable. When a concept dealing with gradual matters does not mention degrees, one may justifiably suspect that the concept misrepresents reality. For example, dispositions are talked about as if the matter were one of all or nothing.

Furthermore, it seems that there is no theoretical benefit to be gained by framing principles and definitions in terms of dichotomies instead of degrees. In fact, finding dichotomies is not a necessary condition of meaningfulness nor of intelligibility. Coherence does not depend on there being a sharp division. If reference to degrees cannot be eliminated, then the need to make dichotomic classifications is neither a methodological imperative nor a requirement of reason. Rather they will appear to be false, or fictitious. Problems become less strenuous the moment one relieves oneself from the demand to draw a neat dividing line. Dichotomies do not have more explanatory power either. A correlation between gradual properties is more illuminating and superior than a low grade correlation of kinds. To illustrate the point, think of how freedom and responsibility are related to each other: one is held

14 Leibniz enunciated his Principle of Continuity thus:

> Nothing takes place suddenly, and ... *nature never makes leaps*. ... It implies that any change from small to large, or vice versa, passes through something which is, in respect of degrees as well as of parts, in between... [par le mediocre... En juger autrement, c'est peu connoistre l'immense subtilité des choses qui enveloppe un infini actuel tousjours et partout] (Rescher 1991: 73; [Finster et al. (comp.), 55-6]).

responsible to the extent that one was free to have done otherwise. One's degree of responsibility diminishes as the degree of freedom decreases.

On the other hand, even if a dichotomy is driven between two opposite properties, the degrees of each do not thereby disappear. For instance, regardless of where the line between rich and poor is drawn, the degrees of being poor and rich remain unaffected.

To end this review, it is worthwhile mentioning that while the progressive extents of a property are perceptible, the crisp boundary that supposedly marks the extension of a predicate is nowhere noticeable.

Needless to say, we completely agree on the points here summarized. We epitomize the main ideas advanced. The existence of distinct degrees of possession of a property *F* is manifested by the different extents to which it is instantiated. Degrees of a property make possible a smooth change. Consequently, a definition of '*F*' neglecting degrees distorts *F*. And linguistic competence in using '*F*' must include the ability to recognize to what degree *F* is exemplified.

2.- Goguen

The earliest detailed attempt to cope with the sorites argument using a fuzzy logic was made by J.A. Goguen [1968-69]. In this section I present his way of resolving the paradox. Goguen's intention is to develop a model of our use of fuzzy words.

Let me begin by exposing the sorites reasoning in terms of the fuzzy set theory. He symbolizes the set of short people by 'S'. S is a proper subset of the set X of human beings, i.e. $S \subseteq X$. The first premise of the inference affirms that there are short people. This means that S is not empty. The major premise states that, for any pair of subjects, x and y, if the difference in height between them is beyond our power of discernment -say, less than one millimetre-, and x is short, then y is also short. In symbols,

(1) If $h(y) - h(x) < 10^{-3}$ and Sx, then Sy, for all x and y belonging to X,

where *h* is a function mapping individuals from X into the set of positive real numbers **R**. For example, *h* correlates Peter with the number 1.79, representing Peter's height in metres. To express what type of function *h* is, customarily we write $h: X \rightarrow \mathbf{R}$. And by repeated application of *Modus Ponens*, we arrive at the conclusion that everybody is short.

Now, were we to persist in keeping the validity of the argument, as classical logicians do, we should give up the weakest premise, (1). By the Principle of Excluded Middle, the falsity of (1) would entail that its negation is true, namely:

(2) $h(y) - h(x) < 10^{-3}$ and Sx, but $\neg$Sy, for some x and y belonging to X.

But (2) is less plausible than (1). Therefore, either the PEM or MP are not valid when the reasoning contains a fuzzy expression. Classical logic, then, is not an adequate theory to represent fuzzy language.

Goguen's solution of the paradox incorporates the following elements: a membership function for the predicate 'short', an interpretation of the conditional premises, and the definition of valid argument. Additionally, a subsidiary notion of a non standard conjunction will be required. Let us examine each component.

2a.- Semantics of the Predicate 'Short'

First, it is assumed that whether somebody is short will depend only on her height; shortness as applied to people is a function of height. That is, $Sx = f(h(x))$. We saw that $h(\text{Peter}) = 1.79$. This numerical value that is Peter's height becomes the input of the function *f*, which maps it into the closed unit interval, [0, 1]. In turn, this second number taken from [0, 1] constitutes the truth value of the original affirmation that Peter is short, or that he belongs to the set of short people, and measures the degree to which 1.79 is a short height. The unit interval is the set of truth values. Yet, 1 alone is the unique value that is true (maximalism), and only 0 is false (333)[15].

What output *f* returns for each input depends on the particular shape *f* takes. Goguen avows that there are many functions that represent 'short', varying with user and context (331). In his opinion, what is meaningful is not much the exact numerical values of shortness but their ordering (332).

In any case, beside being of the type: $\mathbf{R} \rightarrow [0, 1]$, *f* must meet some extra constrains to properly play its role of being the referent of the fuzzy predicate 'short' (331). If there are some short people, then 'Sx' should take the value 1, or a fairly large number. Moreover, *f* must be continuous and monotone decreasing. That *f* is *continuous* means that a small change in the input does not produce a large change in the output (Klir and Yuan: 63, 310). Since changes in height are smooth, so are changes in shortness. The «continuity of *f* insures that S has a smooth boundary» (331). Notice that it is the unit interval's being the range of the functional values of *f* what makes it possible that S has a fuzzy boundary. And that *f* is *monotone decreasing* wants to say that an increase in the input does not yield an increase in the output (Klir and Yuan: 52). This requirement fits our judgement that, in as long as *h*(x) gets bigger, the taller a person is, and the less short she is (331). Finally, *f* shall be a function that asymptotically approaches zero as the height increases (*Ibid.*).

There is a little worry about the last condition. It has the effect that no matter how tall a person is, she will be somewhat short. Though Goguen does not draw the next consequence, it seems to follow from the last constraint that everybody would be short. In fact, that /Sx/ = 0 means that 'Sx' is false (333), or that x is not short *at all* (331). But, by the last condition on *f*, 'short' is never false of anybody, /Sx/ being different from 0 for any x, we can assert shortness of anybody. Hence, 'Sx' would be true of everybody. However, Goguen will object that from the fact that 'p' is not false, we cannot conclude that 'p' is true, for the intermediate values are indeterminate (336). Besides, as a consequence of his maximalist stand, the principle of excluded middle fails, for it may take a value different from 1 (340, 359). So, '∀xSx' will not be false, nor true, but close to 0.

Thus, «any... function» that is continuous, monotone decreasing and asymptotic to zero will provide a representation of 'short', and it is good enough to avoid the paradox (331, 333).

In brief, a fuzzy expression, '*F*', will be represented by a fuzzy set. And a *fuzzy set* is defined as a function mapping individuals from the universe into the unit interval.

2b.- Connectives

The second component of Goguen's treatment of the sorites is his understanding of the conditional (352-3, 356). This is a kind of implication, governed by the following rule:

$$/p \Rightarrow q/ = \begin{cases} 1 \text{ if } /q/ \geq /p/; \\ /q/ \div /p/, \text{ otherwise.} \end{cases}$$

The value of the implication is 1 whenever the value of the consequent is greater than or equal to the value of the antecedent; and otherwise it is the ratio of /q/ to /p/. The next table shows the truth values of Goguen's implication for a pentavalent logic.

15 Two additional manifestations of maximalism is his definition of tautology (365) by appealing to value 1, and his association of tall men with definitely not short people (331).

⇒	1	¾	½	¼	0
1	1	¾	½	¼	0
¾	1	1	⅔	⅓	0
½	1	1	1	½	0
¼	1	1	1	1	0
0	1	1	1	1	1

In order to grasp the rationale behind the assignment of values for all cases where /q/ < /p/, inspect the fifth column of the matrix, the one headed by ¼, leaving aside the bottom row corresponding to the case where /p/ = 0. We notice that the value of the connective constantly increases from ¼ to 1, i.e. approaches truth, according as the difference between antecedent and consequent becomes narrower. In general, the less /q/ descends below /p/, the larger /p⇒q/ is. Conversely, the implication approaches zero inasmuch as the value of q distances itself more and more from the value of p; yet, in the cases we are considering, when /q/ < /p/, the value of the implication never goes below the value of the consequent.

So, the major premises of the sorites will be interpreted as implications, which will be symbolized by '⇒'.

Concerning negation, Goguen's logical system uses a strong negation, defined as '¬p' = p⇒0. That 'p' is false means that p implies zero (358-9, 362).

Finally, keep in mind that Goguen does not use the ordinary conjunction, but instead the arithmetic product, or simple multiplication, symbolized by '·' (347).

2c.- Gradual Validity

Third, Goguen clarifies the essential notion of valid argument by establishing the traditional relation between valid deduction and true corresponding conditional, namely: the inference from "p" to "q" is valid iff the implication "p⇒q" is true (356). For Goguen, when the argument is composed of several premises, it seems -though the matter is not so clear- that the antecedent of the conditional mirroring the reasoning will consist of the conjunction of the premises. Since I have been unable to find evidence to the contrary, I will suppose that to get the truth value of the conjunction, one has to multiply the values of the premises (Cfr. 335-6, 352). Given that, in the logic of inexact concepts, the implication is true when it gets value 1, *valid inferences* are those in which the value of the conclusion is at least as high as the conjunction of the premises. From this conception of validity, it results that the measure of validity of an argument is given by the truth value of the implication reflecting the deduction (335).

2d.- How does the Sorites Function?

Having all the necessary tools at our disposal, let us examine what happens to the sorites.

We can grant that the first premise, that there is a short man, is evidently true. Let us suppose that 'Sa_0' has value 1.

The second premise is: $Sa_0 \Rightarrow Sa_1$. What truth value does this first instance of the major premise get? Since the apparent heights of a_0 and a_1 are indiscriminable, we might think that the fact that a_0 is short does imply that a_1 is also short. However, note that, as the height of a_1 surpasses that of a_0, a_1's shortness must decrease, for, as the height of a person increases, her shortness decreases. Consequently, because a_1 is less short than a_0, the truth value of the consequent of the implicational premise must be a little bit smaller than the value of its antecedent. Thus, '$Sa_0 \Rightarrow Sa_1$' cannot be true, being less than 1.

What exact truth value this major premise receives will depend on the choice of the function denoted by 'short', and the number of individuals in the series. For the sake of simplicity, let us assume that we have 1001 individuals, the difference in height between any consecutive members being of 1 millimetre. a_0 is 1 metre high, a_{1000} is 2 metres high. a_0 is short to degree 1, and a_{999} is short to degree 0.001. a_{1000} will be short to degree less than 0.001, but greater than 0, due to the last condition on the function representing 'short'. So, the envisioned membership function corresponding to 'short' is a straight line -not a curve- descending from the value 1 of a_0 until the value 0.001 of a_{999}. And from there, it becomes an asymptote. Perhaps this is not the best choice, but let us not make a fuss about this. If we concede that these assumptions are not problematic, then $/Sa_0/ = 1$, and $/Sa_1/ = 0.999$.

Now, in view of the permanently smaller degree of the consequent with respect to the antecedent, the truth value of all implicational major premises of the sorites is the quotient obtained by dividing /q/ by /p/. Thus,

$$/Sa_0 \Rightarrow Sa_1/ = 0.999 \div 1 = 0.999.$$

Concerning the value of the second major premise, we have that:

$$/Sa_1 \Rightarrow Sa_2/ = 0.998 \div 0.999 = 0.998, \text{ with '998' periodic.}$$

And so on. Each subsequent major premise receives a truth value slightly smaller than that of the preceding implication. By the time we arrive at the penultimate implication, we discover that:

$$/Sa_{998} \Rightarrow Sa_{999}/ = 0.001 \div 0.002 = 0.5.$$

The truth value of the premise has decreased considerably, but has not gone below the middle point. The same interpretation concerning the lowest value of the implicational premise is reached by Paoli (2003: 367).

As for the last premise, $Sa_{999} \Rightarrow Sa_{1000}$, it contains an antecedent and a consequent none of which are zero, or false. As a result, its evaluation will depend on the value of the apodosis; more concretely, on how far the value of 'Sa_{1000}' will descend below the value of 'Sa_{999}'. Let me illustrate the case with two possibilities.

$$/Sa_{999} \Rightarrow Sa_{1000}/ = 0.00099 \div 0.001 = 0.99$$
$$/Sa_{999} \Rightarrow Sa_{1000}/ = 0.00001 \div 0.001 = 0.01$$

So, there is a wide range of variation for the truth value of the last implication. '$Sa_{999} \Rightarrow Sa_{1000}$' is neither 1 nor 0; i.e., neither true nor false, as all the other major premises. Fortunately, in order to realize what is going on in the whole inferential process, we do not need to know the precise value of the last conditional.

What determines the validity of the entire argument is whether the truth value of the conclusion, Sa_{1000}, is at least as great as the conjunction of the premises, that is, whether $/Sa_{1000}/$ is equal to or greater than the product of the truth values of all premises. The presumption is that the argument is not valid. Let me quote Goguen. He announces that he will «propose a different representation of 'short' which avoids the paradox by rendering the deduction on which it is based invalid» (330). There is further evidence in the same sense: «...the product of a long chain of only slightly unreliable deductions can be very unreliable» (327). The deductive process of the sorites «becomes less and less valid as the number of applications of *modus ponens* increases» (335). And when one calculates the value of the implication by dividing /q/ by /p/ for the cases in which the consequent drops below the antecedent, «the validity of a chain of nearly valid deductions decreases as the length of the chain increases» (352). So, it seems that Goguen expects an affirmative answer to the question of whether the value of the conclusion drops below the value of the conjunction of the premises.

Notwithstanding, I am unable to see the reason for Goguen's expectation. Let me show why I think that the whole chain seems to be valid according to the assumptions made above. The next table lists the sequence of sentences composing the entire sorites with an indication of their truth values. The extreme right column shows the validity measure of each application of modus ponens.

Value of:	Atomic sentence	Major Premise	Validity of Partial Deduction
Sa_0	1		
$Sa_0 \Rightarrow Sa_1$		.999	
Sa_1	.999		1
$Sa_1 \Rightarrow Sa_2$		.99899899	
Sa_2	.998		1
$Sa_2 \Rightarrow Sa_3$		.99899799	
Sa_3	.997		1
:			
:			
Sa_{997}	.003		1
$Sa_{997} \Rightarrow Sa_{998}$		.666666666	
Sa_{998}	.002		1
$Sa_{998} \Rightarrow Sa_{999}$		.5	
Sa_{999}	.001		1
$Sa_{999} \Rightarrow Sa_{1000}$		?	

Sa_{1000}	<.001 but >0		1

Is the first application of implicational modus ponens valid? It seems so. The case can be generalized. Each application of modus ponens must be evaluated according to the definition of validity:

$$'Sa_i,\ Sa_i \Rightarrow Sa_{i+1} \vdash Sa_{i+1}' \text{ is valid iff } /Sa_i/ \cdot /Sa_i \Rightarrow Sa_{i+1}/ \leq /Sa_{i+1}/$$

taking the dot, '·', as the multiplication sign. Let me abbreviate the truth values of antecedent and consequent by a and b, respectively, and let us use the slash, '/' to signify division, instead of the previous '÷'. Since the truth value of the consequent of each implicational premise is always smaller than that of its antecedent, the value of the product of the premises is calculated by:

$$a \cdot b/a$$

which will obviously be equal to the value of the consequent, b. Therefore, each instance of modus ponens, having value 1, is valid.

And a similar outcome is applicable to the entire reasoning. The result of multiplying the truth values of all the premises taking part in the chain is always less that the value of the conclusion. In fact, denoting the truth value of 'Sa_i' by a_i, the product of the values of the premises is:

$$a_0 \cdot a_0 \Rightarrow a_1 \cdot a_1 \cdot a_1 \Rightarrow a_2 \cdot a_2 \cdot a_2 \Rightarrow a_3 \cdot a_3 \cdot \ \ldots\ \cdot a_{998} \cdot a_{998} \Rightarrow a_{999} \cdot a_{999} \cdot a_{999} \Rightarrow a_{1000}$$

Since the value of each implication is calculated by dividing a_{i+1} by a_i, we have:

$$a_0 \cdot a_1/a_0 \cdot a_1 \cdot a_2/a_1 \cdot a_2 \cdot a_3/a_2 \cdot a_3 \cdot \ldots \cdot a_{998} \cdot a_{999}/a_{998} \cdot a_{999} \cdot a_{1000}/a_{999}$$

We see that, in each instance of modus ponens, the value of the minor premise reappears as denominator of the value of the major premise. Cancelling them out, we obtain:

$$a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_{999} \cdot a_{1000}$$

Because all these factors are less than 1, the result of multiplying them will be smaller than the last factor, a_{1000}. Hence, the value of the conclusion of the chain is greater than the value of the product-conjunction of the premises. Therefore, against Goguen's diagnosis, the whole argument is valid, since its corresponding implication has value 1.

My explanation of Goguen's error -if it is one- is that he has mistaken the value of each n^{th} major premise as the measure of the validity of the n^{th} sub-deduction.

Remark that, from a contradictorial gradualism, we can accept that there is a valid sorites concluding that everybody is short, but employing the material conditional for the implication. For more on this, see Chapter 1, §§ 4e and 6; and Chapter 6, § 6d.

2e.- Conclusion

I conclude that, unless my construal is wrong, by reading into Goguen's theory some unwarranted assumption, he avoids the paradox by declaring it unsound: the argument is valid, but no premise is true, except the first. And on these two particular points, we somehow agree with him. If we deal with an unbounded series, where it is always possible for persons to be taller and taller, and the major premises are implications, then the rule of inference, implicational *modus ponens* is valid, but no major premise is true at all.

Additionally, we share his opinion that, at the moment of representing fuzzy properties, classical logic is inappropriate, fuzzy sets doing a better job in this respect. The set of truth values should consequently be enlarged to make it possible the continuity of the boundaries of fuzzy properties.

However, we disapprove his maximalism, ensuing in the failure of the law of excluded middle and of the weak principle of bivalence (indeterminism).

3.- Lakoff

George Lakoff [1973] is another pioneer of the use of fuzzy sets to account for fuzziness of natural language, specially the semantics of those particles that are known as hedges, such as 'very', 'sort of', 'more or less', 'rather', 'strictly/loosely speaking', 'somewhat', etc. Intuitively, *hedges* are «words whose job is to make things fuzzier or less fuzzy» (471). Lakoff's chief claim is that hedges can only be described in the frame of fuzzy set theory, but not in a two-valued system.

In order to make his case, he refers to the work done by Eleanor Rosch, who has made empirical research to determine whether category membership is a clear-cut, yes-no issue or a matter of degree. Her findings establish that the categories studied admit of central and peripheral members ranked in a hierarchy. For example, for the category of 'birdiness', there is the following hierarchy: robins / eagles / chickens, ducks, geese / penguins, pelicans / bats. For Lakoff, this internal differentiation within categories is an indisputable fact. So, if the category of bird includes not only its best exemplars, but also other less representative elements, it is quite natural to think of that gradual category in terms of a fuzzy set.

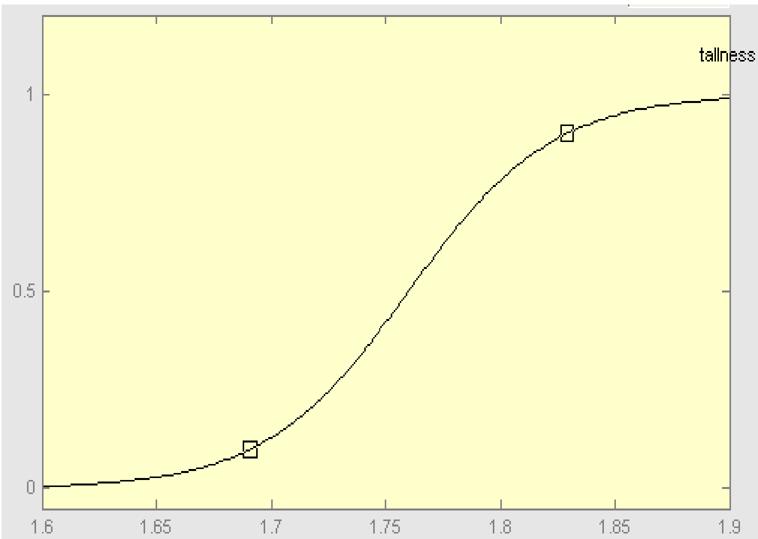
In contrast to a classical set, partitioning the universe of discourse into members and non members, a fuzzy set admits a gradation of members. The *fuzzy set F* is conceived as a set of ordered pairs, $\{<x, \mu_F(x)>\}$, where '$\mu_F(x)$' is a membership function indicating the degree to which x belongs to *F*. The value μ_F assigned to an individual x depends on x's properties, which in turn may vary by degrees. Thus, μ_F itself may be a function of other membership functions: $\mu_F = f(\mu_{G_1}, \dots, \mu_{G_k})$. The right member of this equality is called the '*vector value*' of *F*. Each of the μ_{G_i} is a meaning component, or criterion of '*F*'.

One of the examples analysed by Lakoff is the property tall. Clearly, tallness depends on height. The question is how tall an individual has to be to be tall, say, in contemporary American society. Lakoff thinks that there is no single, fixed answer to this query (458). A subjective approximation of the membership function relative to the mentioned context is displayed in the following chart, where I have substituted metres for feet of the original, rounding up the quantities.

Height in metres	1.6	1.65	1.7	1.75	1.8	1.85	1.9
Degrees of tallness	0	0.1	0.3	0.55	0.8	0.95	1

These values mean that a person measuring less than 1.6 m is not tall «to any degree» (462), while one being 1.9 m high is tall, period. Somebody 1.7 m high is tall to degree 0.3. Though the exact values should not be taken with all seriousness (*Ibid*.), Lakoff says that the function has about the right shape. The curve raises continuously.

Fig. 1



From the fact of gradual membership, Lakoff immediately jumps to the conclusion that there are degrees of truth. For, if x belongs to set *F* only to a degree, Δ, then the sentence that 'x is *F*' should be true to that degree Δ (460). In other words, the truth value of 'x is *F*' is the degree of membership of the object denoted by 'x' in the set *F* (466). Thus, most people confirm that the sentence 'A robin is a bird' is true, 'A chicken is a bird' and 'A penguin is a bird' are also true, but each sentence less than the preceding one; and 'A bat is a bird' is false, or very far from true, while 'A cow is a bird' is absolutely false.

However, though Lakoff expresses himself as if there were degrees of truth, and falsehood, in a strict sense, I take him as another spokesman of maximalism. Actually, a system can contain intermediate degrees between 0 and 1, and yet, it may be maximalist. Extending the set of truth values beyond bivalence is not a sufficient condition for not being a maximalist. The defining feature of *maximalism* is that the set of designated truth values, D^+, comprises only the unit value, 1. And this is the case with Lakoff: $D^+ = \{1\}$ (Morgan and Pelletier: 82). That this interpretation seems correct can be shown by his assertion that the law of excluded middle, "$p \vee \sim p$", is not a tautology, for, when 'p' is true to degree 0.6, «...one would not want to say that the sentence was true...» (465). Again, when Lakoff says that "$p \wedge \sim p$" is not false, but has a degree of truth, I understand him as simply saying that "$p \wedge \sim p$"

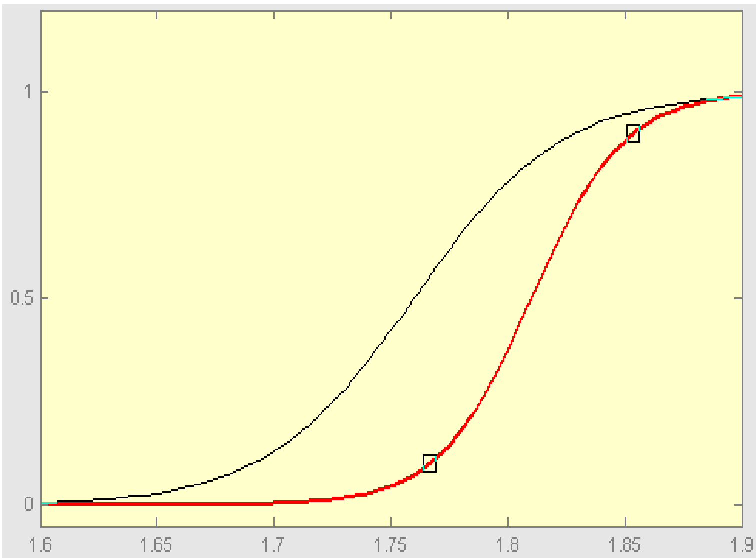
is not 0 (*Ibid*.). Moreover, Lakoff uses the phrase 'close to true' to refer to those values that are near 1 (pp. 471, 484).

3a.- Hedges

As for Lakoff's treatment of hedges, he follows, in some respects, the fundamental insights of the founder of fuzzy set theory, Lofti Zadeh.

Zadeh believes that there is a short number of basic algebraic functions, such as concentration, dilation, and two others, serving to define a large number of hedges. For example, he defines *concentration* as: $\mu_{CON(F)} = \mu_F^{\,2}$, and *dilation*, as: $\mu_{DIL(F)} = \mu_F^{\,1/2}$. These two basic functions have opposite effects: concentration lowers the value of the membership function, and makes the curve steeper, whereas dilation raises the value and makes the curve less steep. If the membership function μ_F on which they operate is bell shaped, then concentration pulls the values of μ_F in, while dilation spreads them out. Again, Zadeh has warned that the use of squares, square roots, etc. is not to be taken seriously, whatever other functions that have more or less the same effects being also acceptable. Zadeh then defines hedges in terms of the functions just described. As an illustration, /very (F)/ = con(/F/). Thus, supposing that Peter is tall to degree 0.9, then he belongs to the set of very tall people to degree $0.9^2 = 0.81$; i.e, the predicate 'very tall' has a degree of membership lower than 'tall'. Figure 2 shows the curves for both, the function corresponding to 'tall', on the left, and that of 'very tall', on the right, according to Zadeh.

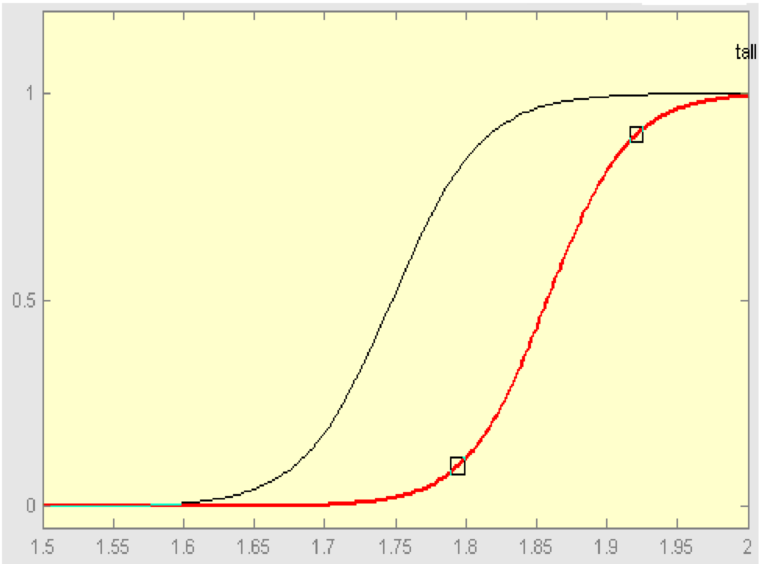
Fig. 2



Lakoff has raised a number of objections to Zadeh's view on hedges, like Zadeh's leaving context out of consideration, and not employing vector values. Another criticism concerns the interpretation of 'very'. Assume that the membership function for 'tall' is the one depicted in the chart above, having values 0 and 1 at heights 1.6, and 1.9 m respectively.

If 'very' were understood as suggested by Zadeh, as having the same effects as the function of concentration, then the points at which it has values 0, and 1, will coincide with those of the function denoted by 'tall'. Yet, for Lakoff, this is counterintuitive, for it would be possible that 'x is tall' has value 1 without 'x is very tall' having that value. For example, James could be 1.9 m high, and therefore be tall to degree 1, but he is not very tall, nor very very tall. To accommodate this, Lakoff maintains that 'very' should shift the values of μ_F to the right, as shown on the following figure.

Fig. 3



Despite these disagreements, Lakoff concludes that the core of Zadeh's proposal remains sound. A proper understanding of hedges requires that the sentences that they are attached to admit degrees of truth. This being so, an adequate description of the modifiers lies outside the scope of classical logic.

3b.- Assessment

We believe that some fundamental theses of Lakoff are adequate. Among them, I mention the following. Because categories like 'bird' display an internal hierarchy, including best exemplars and peripheral members, it seems natural to represent those categories by means of fuzzy sets, allowing a gradation of membership. In turn, this gradual membership entails degrees of truth. And finally, a hedge such as 'very' is best modelled within a framework using degrees of truth, since that hedge diminishes the value of the property it affects.

Nonetheless, we take exception to Lakoff's maximalism and to his abandoning the principle of excluded middle. For a discussion of maximalism, see Chapter 1, § 6, and Chapter 6, § 6d.

4.- Machina

Kenton Machina is the most famous representative of the infinitely-valued approach to fuzziness, upholding also some paraconsistent theses. In his article [1976], he motivates the philosophical foundations for a logic intended to capture fuzziness. Since he is a required point of reference, we expose in this section the main themes of his theory.

4a.- The Philosophical Understanding of Vagueness

First, Machina is of the opinion that classical logic works good for exact propositions but not for vague ones, the reason for this being that the *Principle of Bivalence* is in conflict with vagueness. That principle is understood as demanding that every proposition be simply true or simply false, but not both. But the PB fails because there are circumstances in which a sentence is neither simply true nor simply false (49). Indeed, Machina thinks that vagueness issues in indeterminacy as to the truth conditions of a sentence, i.e., the range of possible facts that will verify it is somewhat indeterminate.

However, though bivalence is abandoned, to deprive a sentence from having a truth value is also prohibited. Machina opposes indeterminism, arguing that it is incompatible with the Tarski convention T. The reasoning is as follows.

(1) 'p' lacks any truth value	Indeterminist thesis
(2) 'p' is true ≡ p	Tarski Convention T
(3) "p' is true' is false	(1)
(4) p is false	(2), (3), subst.

The incompatibility between (1) and (4) would show that (2) has to go, if (1) held. Yet, Tarski Convention T is completely true (75). Ergo, sentences should not be allowed to be indeterminate in respect of truth value.

Thus, vagueness requires that classical logic be extended, so that sentences may receive truth values different from the standard ones. The intermediate values are construed as degrees of truth, where 'T', and 'F' are interpreted as completely true and completely false, respectively.

As an illustration of a vague situation, Machina (54-5; 58) refers to Horatio's planting petunias in the garden yesterday, in such a way that he simply lays the baby plants on top of the soil, which then is watered. The set of actions constituting Horatio's planting petunias is neither a full-fledged, clear example of planting petunias, nor is it a clear, full-fledged example of failing to plant petunias, as if he had dumped the baby plants into the garbage can and had gone to the beach. In *Aj*, we can symbolize the situation as follows:

$$\sim Hp \wedge \sim\neg p$$

Rather, the case is one such that Horatio to some extent succeeded in planting petunias and to a certain extent he failed to plant petunias:

$$Lp \wedge L{\sim}p$$

That is, to a certain extent he planted and did not plant petunias:

$$L\,(p\wedge{\sim}p)$$

Thus, a vague situation is contradictory. For, if Horatio plants petunias but to such a degree that he does not completely exemplify petunias planting, to that degree Horatio does not exemplify petunias planting:

$$Lp \wedge {\sim}Hp \supset L{\sim}p$$

So, it is not that Horatio simply failed to plant petunias but came close to it. If Horatio sort of planted petunias, he really somewhat planted petunias:

$$Lp \supset HLp$$

The last formula displayed says that, if a sentence is true to some degree, then it is absolutely true that the sentence is true to some degree. In other words, if it is true that 'p' is more or less true, then, to say «'p' is more or less true» is completely true (Peña 1991: 77; theorem number 302/2 of *Aj*). So, it is not that a vague situation is indeterminate.

The previous example makes it manifest that an object *a* can exemplify a property *F* without doing it to the utmost degree. And similarly, *a* can to some extent fail to exemplify *F*, but without completely failing to exemplify it. Both, to possess and to lack a property are a matter of degree.

And exactly the same conditions are replicated at the level of a sentence's truth. Indeed, if "p" is true to some degree but not completely, then, "p" is false to some extent, but again, not completely. Therefore, if "p" is neither totally true nor totally false, it is bound to be contradictory: true up to a point, and false up to a point:

> it now seems to be an essential characteristic of a vague proposition that a contradiction... can be partially true (59).

And the same applies for propositional attitudes, like Jones' belief that Horatio planted petunias in the garden yesterday: it is partially true and partially false.

4b.- The Logic of Vagueness

On the basis of the previous philosophical conception of vagueness as gradual and contradictory, Machina sets up his logic of vagueness. The construction of the logical system must obey certain constraints. First, the logic has to be *normal*: when the only truth values taken into consideration are the classical ones, then one gets exactly the same consequences as in classical logic; that is, all its tautologies and rules of inference shall be preserved in the logic of vagueness. This implies that, for example, the negation of a true sentence is not something different from a false sentence. And the truth-functionality of the logical connectives is also kept.

But what about the principles of excluded middle, $p\vee\sim p$, and of non contradiction, $\sim(p\wedge\sim p)$? Does not vagueness force the abandonment of these truths? The principles are lost only in the sense that they are not absolutely true. Though in this sense they fail, there is another sense in which they are preserved, for none is utterly false, but only at most 50% false. In point of fact, both laws are at least half true. It is worthwhile to consider the case of the Principle of Non Contradiction. When 'p' is a sentence whose truth value is 0.5, its negation, '$\sim$p', is also true to that same degree, and therefore, their conjunction, $p\wedge\sim p$, is 0.5 true too, for conjunction is truth-functional, as all other connectives, and each conjunct is partially true. In all other cases, to the extent that one of the conjuncts is truer than 50%, to that extent the principle of non contradiction is more than 50% true, and to that extent, the contradiction is more than 50% false. In this manner, *penumbral connections*, i.e., logical relations holding among vague sentences, are not renounced; rather they are real.

Underlying these considerations is the thought that the negation of a sentence inverts the truth value of the affirmative sentence. That is, the truth value of "$\sim$p" is calculated by subtracting from 1 the truth degree of "p". Symbolically, $/\sim p/ = 1 - /p/$. As a result, the truer a sentence is, the falser its negation is (57). In other words, as truth increases, falsity decreases. And vice versa. In symbols, $/p/ > /q/$ iff $/\sim p/ < /\sim q/$.

Concerning conjunction and disjunction, they take the minimum and the maximum values of their respective members. I.e., $/p\wedge q/ = \min(/p/, /q/)$; and $/p\vee q/ = \max(/p/, /q/)$.

As for the conditional, Machina wants to keep a close link between the validity of the argument from "p" to "q" and the truth of the corresponding conditional, "if p, then q". The properties bestowed on the conditional make it a sort of implication, as it is construed in *Aj*; hence we will reserve for it the symbol '$\Rightarrow$'. Whenever the value of the consequent is greater than the value of the antecedent, one has that $/p\Rightarrow q/ = 1$. He also wants that all self-implications, "$p\Rightarrow p$", be totally true. And for this reason, he excludes the definition of the conditional as "$\sim p\vee q$", for in this case, the principle of self-implication would be less than completely true when "p" bears an intermediate truth value. In the remaining cases, when the apodosis is falser than the protasis, the implication will be somewhat false. The measure of the falsity of the implication will depend on how much the truth value of "q" dips below the value of "p": the more the consequent distances itself from the antecedent, the greater the falsity of the implication, or the less true "$p\Rightarrow q$" is. All these desiderata are captured in the following rule.

$$/p \Rightarrow q/ = \begin{cases} 1, \text{ if } /q/ > /p/ \\ 1 - /p/ + /q/, \text{ if } /q/ \leq /p/. \end{cases}$$

An instance of this implication for a pentavalent logic is displayed in the next matrix.

⇒	1	¾	½	¼	0
1	1	¾	½	¼	0
¾	1	1	¾	½	¼
½	1	1	1	¾	½
¼	1	1	1	1	¾
0	1	1	1	1	1

The truth values of the equivalence are taken from the usual definition: $/p \Leftrightarrow q/ = /p \Rightarrow q \wedge. q \Rightarrow p/$.

Associated with this characterization of the implication is Machina's rejection of the usual notion of validity as designation preserving argument form, according to which a designated value of the conjunction of the premises guarantees a designated value of the conclusion. Rather, the definition of a *valid argument form* more in line with his conception of the implication is that an argument form is truth preserving iff the truth value of "q" is at least as high as that of "p". In symbols: $p \vdash q \Leftrightarrow /q/ \geq /p/$. This means that in order for the conclusion to [strictly] follow from the premises, it must be as true as, or truer than the falsest of the premises.

Machina claims that the generalization of this definition is the notion that "*p implies q to degree n*", $p \vdash_n q$, for a value of *n* in [0, 1]. Without being rigorous, this is explained in the following manner. If the value of "q" is at least as true as the value of "p", then p implies q to degree 1, $p \vdash_1 q$. This is full validity. There is also less than full validity. The amount of decrease from validity to degree 1 of $p \vdash q$ is determined by how much the degree of truth of "q" can descend below the degree of truth of "p". And vice versa, if "p" implies "q" to degree *n*, then the truth value of "q" can fall below the value of "p" as far as 1-*n*. For example, supposing that a given set of premises implies to degree 0.9 a certain conclusion q, then the value of q can drop below the value of the falsest of the premises by at most 0.1. Thus, if the falsest premise is 0.3 true, then the conclusion cannot be less than 0.2 true. Being more precise, that "p" implies "q" to degree *n* means that *n* is the least upper bound of the values *m* such that $(/p/ - /q/) \leq (1\text{-}m)$, for all assignments of values to "p" and "q", provided that, for at least some of these values, the result of the difference /p/-/q/ is positive.

It is important to note that, according to this definition, "p" utterly fails to imply "q", that is, the argument from "p" to "q" is absolutely *invalid*, only if it is possible for "q" to be completely false even though "p" is totally true (71).

As the set of truth values, Machina chooses the real numbers in the unit interval (60). One ground for this selection is that it is conceivable that there is a continuum of borderline cases, each exemplifying the property *F* to a different degree, so that it would be arbitrary to identify the truth values of 'Fa_i' and 'Fa_{i+1}'. This suggests a continuum of truth values. Were there only three truth values, one could not accurately represent the series of the borderline cases ordered by their degree of possessing *F*.

A predicate letter '*F*', of order *n*, will be assigned a *fuzzy set*, which is nothing but a function mapping a tuple of *n* individuals into the index set. In the monadic case, the value assigned by the fuzzy set to the individual *a* represents the degree to which *a* possesses the property *F*. In order to provide a natural connection with the previous choice of the unit interval as the truth set, Machina also identifies the index set with the [0, 1] interval, though he clarifies that this move is not necessary (65).

The rules governing the semantics of the quantified sentences are that '$\forall xp$' and '$\exists xp$' take, respectively, the greatest lower bound, and the least upper bound, of the truth values of all possible instances resulting from replacing the names of individuals in the domain for the bounded variable 'x'.

4c.- Explaining the Plausibility of the Sorites

Now that we have all the necessary logical apparatus, we can scrutinize the sorites. Let me begin by presenting the first steps of the reasoning chain, and then comment on each partial inference. In what follows, 'Nx' stands for the fact that x has no hair on his scalp; 'Mxy' means that x has one more hair on his scalp than y; 'Bx' symbolizes the fact that x is bald; and 'h' stands for Horatio.

(1)	Nh
(2)	$\forall x\ (Nx \Rightarrow Bx)$
(3)	$\therefore Bh$
(4)	$\forall x \forall y\ (Mxy \wedge By \Rightarrow Bx)$
(5)	$\therefore \forall x\ (Mxh \Rightarrow Bx)$
(4)	$\forall x \forall y\ (Mxy \wedge By \Rightarrow Bx)$
(6)	$\therefore \forall x \forall y\ (Mxy \wedge Myh \Rightarrow Bx)$
(4)	$\forall x \forall y\ (Mxy \wedge By \Rightarrow Bx)$
(7)	$\therefore \forall x \forall y \forall z\ (Mxy \wedge Myz \wedge Mzh \Rightarrow Bx)$

Let us examine the argument from (1) to (3). As a matter of fact, the first two premises are completely true, and so the first conclusion is also absolutely true. Yet, the form of the argument is not fully valid, for it is not truth preserving. Indeed, let us assume that /(1)/ = .6, and /(2)/ = .4. Under these assumptions, we can calculate the truth value of the first sub-conclusion, /(3)/. The maximum amount of decrease of the truth value of a sub-conclusion with respect to that of its first premise is limited by the falsity of its second premise. That is,

$$(/(1)/ - /(3)/) \leq (1 - /(2)/)$$

And from this, we obtain

$$/(3)/ \geq .6 + .4 - 1$$
$$/(3)/ \geq 0$$

Since the truth value of the first sub-conclusion may be 0, it means that, from premises somewhat true, the first sub-argument can lead to a completely false conclusion. Nonetheless, Machina asserts that it is unfair to simply say that the argument form is invalid, for the argument has some validity.

In general, with arguments consisting of two premises whose truth values are, respectively, m and n, the value of the conclusion is at least $m+n-1$ true. And only to this degree the argument form is valid. If the sum of the truth values of the premises is close to 2, then the argument validity is close to 1. In order for the conclusion to have a value greater than 0, the sum $m+n$ must be greater than 1.

The sorites continues to have more problems with its inductive premise, (4): $\forall x \forall y (Mxy \wedge By \Rightarrow Bx)$, that any person having one more hair than a bald person is also bald, due to its not being completely true. Indeed, the truth value of (4) is the greatest lower bound of the values of all its instances, namely:

$$(8)\ Mxy \wedge By \Rightarrow Bx$$

Machina holds that the relation M, of having one more hair than, is classical, and that it is true that x has one more hair than y. Hence 'Mxy' is 1. From this, it results that /(8)/ reduces to /By⇒Bx/. Machina believes that, in order to evaluate (8), there are two kinds of cases to consider. First, the cases in which x and y are completely bald, though x has one more hair than y. Here, both sentences, 'By' and 'Bx' are completely true, and hence, /By⇒Bx/ = 1, which is also the truth value of (8), for this first case. And second, in the remaining cases, it happens that a difference of one hair between individuals bearing the relation M, of having one more hair than, makes them almost equally bald, but not exactly bald. Then, x will be a little bit less bald than y, and therefore, the truth value of 'Bx' will dip a little bit below that of 'By' (73). The difference between the truth values of both sentences will be a very, very small fraction, say, in the order of 10^{-5}. So, let us suppose that /By/-/Bx/ = ε. Since the value of 'Bx' is less than that of 'By', the truth value of 'By⇒Bx' will be less than 1, its degree of falsity being equal to the amount by which /Bx/ descends below /By/. Consequently, /By⇒Bx/ = 1-ε. Hence, in this second kind of cases, the value of (8) is 1-ε. Gathering the valuations in the two kinds of cases, we get that /(8)/ is either 1 or 1-ε. And because it is impossible for (8) to be less than 1-ε true, it follows that the universal quantification of (8), i.e. (4), is the greatest lower bound of both possibilities. Thus, /(4)/ = 1-ε: the major premise of the sorites is not completely true, but it is very true.

Let us pass to the second sub-argument, from (3) to (5). In order to ascertain the truth value of its conclusion, we repeat the same procedure employed to discover the value of the first sub-conclusion. From the basic relation:

$$(/(3)/ - /(5)/) \leq (1 - /(4)/)$$

we deduce that

$$/(5)/ \geq (/(3)/ + /(4)/ - 1)$$
$$/(5)/ \geq /(4)/$$

for /(3)/ = 1. So, /(5)/ $\geq$ (1-ε). (5) is at least as true as the falsest of its premises. Hence, the sub-argument from (3) to (5) is fully valid. This result appears fine.

But when we proceed to the third sub-argument, from (5) to (6), by means of (4), we find out that the conclusion drops below the value of the falsest premise. Indeed,

$$/(6)/ \geq (/(5)/ + /(4)/ -1)$$
$$/(6)/ \geq (1-\varepsilon + 1-\varepsilon -1)$$
$$/(6)/ \geq 1-2\varepsilon$$

The degree of validity of this third sub-argument is the value of the implication /(5)/ $\wedge$ /(4)/ ⇒ /(6)/. I.e., 1-ε. So, the guarantee of truth for the conclusion begins to leak away. And this reduction of the degree of truth of the subsequent conclusions persists throughout the sorites. In other words, each time we draw the next consequence, the degree of truth of the sub-conclusion decreases by a margin of ε with respect to the previous sub-conclusion. Thus, the truth value of the sub-conclusion (7) is 1-3ε, and so on. By the time we have gone all the way down, applying the inference pattern 10^5 times, we reach the absurd conclusion that a man having 100000 hairs on his scalp is still bald. One may grant that this conclusion is absolutely false. Yet, the argument is not simply invalid, given that the truth value of the final conclusion dips below the value of its falsest premise by just the small amount ε. This means that, according to Machina's conception of gradual validity, the whole argument implies its conclusion to a very high degree, namely 1-ε. In reality, the degree of validity of the complete argument is determined by the value of the implication: /Γ/ ⇒ 0, where 'Γ' is the conjunction of the truth values of all premises: 1 $\wedge$ 1-ε $\wedge$ 1-2ε $\wedge$ 1-3ε $\wedge$ ... $\wedge$ ε, omitting repeated values. That is, the truth value of the conjunction of the premises is ε. Therefore, the degree of validity of the entire argument is 1-ε. And this is what has happened at each step of the reasoning chain using the premise (4), save the second sub-argument, from (3) to (5), which is valid to degree 1 (p. 74).

Let us summarize Machina's analysis of the sorites. Its major premise (4) is always almost totally true, and in some instances, totally true. But its inference pattern is not fully valid, since, in each sub-argument, the degree of truth of its sub-conclusion persistently falls below the value of its premises by a very small amount. Hence, although each sub-argument implies its conclusion to a quite high degree, the truth of the subsequent conclusions leaks away until none is left. Thus, it is explained why, despite the fact that the reasoning begins with totally true premises, it ends up with a totally false conclusion.

These two characteristics of the sorites, that both the degree of truth of its major premise and the degree of implication of each sub-conclusion from (5) on by the premises are both very high, also explain why the sorites is deceptive. Its major premise and its reasoning pattern appear acceptable, though neither the former is totally true, nor the latter is fully valid. To say that the argument is invalid will not explain why we are so attracted by the chain of reasoning (73).

Machina concludes that his theory has everything one wants. It makes the major premise quite true, seemingly plausible. And the argument form has a high degree of validity, or preserves truth quite well, provided that it is not carried too far. And most importantly, at no step in the chain the guarantee of truth is lost all in one single step.

4d.- Assessment

Machina's conception of fuzziness is quite satisfactory. That an object *a* can possess a fuzzy property *F* to an intermediate degree and that, therefore, *a* can also partially fail to possess *F* are really the heart of the phenomenon we are studying. His accompanying theory of intermediate degrees of truth and of falsity appear a necessary complement to the standard logic, the more so in as long as it allows us to eschew indeterminism. We also totally agree with his admission of partially true contradictions. Again this is a fundamental feature of fuzziness. Concerning Machina's logical system, it is one asset of his point of view to consider that the logic of vagueness should be normal, which allows it to maintain all classical truths. And finally, at least three of the basic tenets of his analysis of the sorites are in the right direction: that the major premise is not totally true, that the argument form is not fully valid, and that in no single step in the argumentative chain we pass from whole truth to perfect falsity.

However we do not coincide in other important points. Especially, I want to discuss his conception of the conditional and of validity of an argument. It is an essential part of the meaning of English compound sentences formed by means of "if..., then" that the truth of the antecedent is sufficient for the truth of the consequent; whenever the antecedent is true, the consequent is also true. This implies that, with a true conditional, it cannot happen that, while its antecedent is more or less true, its consequent is not true at all. In other words, the conditional has to have the modus ponens property. Unfortunately, Machina's logical rendering of the "if..., then" locution flouts this requirement, for it evaluates "$p \Rightarrow q$" as somehow true even though it has a true antecedent but a completely false consequent. We deem this result totally unsatisfying. Perhaps the root of this discrepancy is that it is not enough to demand from the conditional that its degree of falsity be determined by the amount of decrease of the consequent's degree of truth with respect to that of the antecedent; and in fact, this condition is neither sufficient, nor necessary.

Connected with this problem is the one concerning the validity of arguments. Machina wants to have a gradual notion of validity such that, when applied to the appraisal of the sorites, it yields an intermediate assessment, to the effect that the sorites is «not fully valid» (72), nor outright invalid, but something in between. Evidently, he uses the expression "valid to degree n". Indeed, at the moment of assessing the truth value of the first sub-conclusion of the sorites, he offers a formula to calculate it as a function of the values of its premises, namely, $/Bh/ \geq (m + n - 1)$. Commenting on this he says that it «is only to this degree that the argument form is valid in general» (73). We saw that, according to Machina the sorites is only «slightly invalid» (74), or more exactly, it is truth preserving to a very high degree of 1-ε, where ε is a minute quantity. The two great advantages of his treatment is that thereby he explains not only why one can gradually pass from completely true premises to a totally false conclusion, but also why we are deceived by the argument.

Notwithstanding, the surprising outcome is that a reasoning that has true premises to some degree or other may lead to an entirely false conclusion, and nonetheless be such that, according to Machina's definition, its degree of truth preservation is closer to 1 to the extent that the difference between the value of the conjunction of its premises and the value of its conclusion approaches 0. We think that an argument consisting of true premises but with a totally false conclusion should not have a positive degree of truth preservation, for the truth of the premises is completely lost in the consequence. An inference whose premises exhibit some truth that is not preserved at all in the conclusion does not deserve to be called truth preserving. This unwelcome result invites a modification of the truth table for the implication.

5.- Smith

Nicholas Smith has contributed vigorously to our debate defending a new conception of vagueness. In this section I make an exposition of his work. I begin by his innovative definition of vagueness, and then I add two complementary ideas.

5a.- Vagueness as Closeness

In his article [2005b], Smith offers us his characterization of vagueness as closeness, partly in contrast with the view of vagueness as tolerance. Calling '*F* relevant respects' those aspects pertinent to the determination of whether something is *F* or not, 'tolerance' is defined as follows.

Tolerance: if x and y are similar in *F* relevant respects, then '*F*x' is identical with '*F*y' in respect of truth.

If vague predicates were tolerant, then a small change in a respect relevant to the possession of *F* will never produce a change in the truth value of the sentences describing the original and the modified conditions. For example, if *a* is a heap made of 10,000 grains, and *b* is one of 9,999 grains, then both sentences '*a* is a heap' and '*b* is a heap' have the same truth value. The unmistaken insight behind Tolerance is that the removal or the addition of a single grain does not make a huge difference as to whether a pile of grains is a heap or not.

According to Smith, the problem with this principle is that it leads to a contradiction, that every member of a soritical series is and is not *F* (170). Consequently, according to his logic, Tolerance cannot be right. If the removal of one grain from a heap left us still with a heap, then even the removal of all grains would leave us with a heap, which is absurd. And again another incoherence would result with the inverse process of adding grains. In this case, Tolerance has comparable effects to the sum of zero to itself: no matter how many times we add zero to zero, the result will never be a number greater than zero. Analogously, if the addition of a single grain did nothing to contribute to make a difference between what is not a heap and what is a heap, then no matter how many thousands of grains one adds, one would never build a heap. What is wrong with Tolerance is that small differences are not negligible[16]: we do not have the right to ignore them, since, eventually, the aftermath of

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16 As Ernesto Napoli (118) has maintained, it is a mistake to believe that the sum of a small constant is small. The point has also been recognized by Achille Varzi (2003a: 35): small (continued...)

successive removals will be the disappearance of the heap. It might be that one small difference is negligible, but many insignificant differences add up, and issue in a significant difference. Because a negligible difference is a difference, we must take into account the cumulative effect of negligible differences. Therefore, vague predicates are not tolerant, but intolerant. It is not the case that small changes make no difference.

The correct intuition behind Tolerance is that a small difference in *F* relevant respects will never generate a big difference regarding the possession of *F*. But one should not neglect insignificant differences. Thus, we arrive at a new conception of vagueness as closeness, as expressed in the next principle.

Closeness: If x and y are very similar in *F* relevant respects, then '*F*x' and '*F*y' are very similar in respect of truth.

Vague predicates are only those that respect Closeness. Thus, if a_i and a_{i-1} are piles differing merely by one grain, then the truth values of the sentences 'a_i is a heap' and 'a_{i-1} is a heap' will be very similar. In other words, the removal of one grain from a heap will certainly not gives us reason to expect that the truth value of the sentence 'a_i is a heap' is true while 'a_{i-1} is a heap' is false. And vice versa. Contraposing Closeness, we get that the sentences 'Fa_i' and 'Fa_{i-1}' will not be very similar in respect of truth only if a_i and a_{i-1} are not very similar in *F*-relevant respects. So, a considerable disparity in the truth values of the sentences '*a* is a heap' and '*b* is a heap' can only be justified or explained if *a* and *b* widely diverge in the factors determining whether something is a heap.

Notice that this novel characterization of vagueness as Closeness requires the acceptance of gradations of truth. In fact, if a small variation among x and y in *F* relevant respects originates only a small variation in the truth values of the sentences '*F*x' and '*F*y', then it must be possible for there to be two truth values that are distinct and yet very similar (179). It is entirely within the spirit of Smith's theory to suppose that how close the truth values of the sentences '*F*x' and '*F*y' will be depends on how near x and y are concerning the property *F*. This implies that there should be a large supply of truth values that will go hand in hand with arbitrarily small changes in *F* relevant respects. Therefore, in order to accommodate Closeness, a continuum of truth values must be in place. The set of rational numbers will not do, for they lack the completeness property (that every subset has a supremum and an infimum), which is needed for the fuzzy unions and intersections (2001: 108-9).
On the other hand, if the Principle of Bivalence were in force, then Closeness would reduce to Tolerance: to be very close in respect of truth would mean to have exactly the same truth value.

Among the several advantages of Closeness there is this one that it permits to capture the change through a *soritical series*. Smith characterizes this as an ordered sequence of elements such that it begins with an object that is *F*, it ends with an object that is not *F*, and every pair of contiguous members is very similar in *F* relevant respects (170). Actually, it is possible to go from full-fledged *F* to full-fledged not *F* by means of pairs of items that are barely dissimilar (176). Considering the full range of fluctuation of truth values of the sentences resulting from substituting in 'x is *F*' the variable x by the names of every element in the series, in none of the small steps made a sharp boundary is traversed, from true simpliciter to false simpliciter. But every step taken causes a small drop in the truth of the sentence describing the situation, until the last member of the sequence of sentences is reached, which is false simpliciter.

To end characterizing Smith's proposed approach to vagueness, I ought to mention that he thinks that higher order vagueness is part of vagueness. In fact, the motivation for

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(...continued)
differences accrue.

introducing second order vagueness is that for a concept to be vague it is not sufficient to have an intermediate zone of borderline cases. Rather what is required is that this region do not have clear-cut borders, so that there be borderline cases of borderline cases, and so on. Reflecting on this, Smith says that higher order vagueness is nothing else but the demand for a «gradual transition» (173) from the clear cases of the predicate to the borderline ones.

5b.- Degree Functions and Blurry Sets

Now I continue with two additional components of Smith's position found in his [2004]. He presents an original, non-fuzzy degree theory of vagueness (168). Indeed, believing that a vague property can be possessed to intermediate degrees, he introduces two novelties. Instead of modelling fuzzy predicates by standard fuzzy sets, he uses blurry sets. And he maintains that a vague sentence should be assigned as its truth value a degree function in place of a real number from the unit interval, [0, 1].

In fact, the whole motivation for his theory is constituted by his desire to avoid the problem that he sees as the one of higher order vagueness. What is wrong with a logic whose truth values are the real numbers from [0, 1] is the belief that the degree to which a vague property is possessed by an object can be represented by a unique element of [0, 1], all other possibilities being incorrect. It is not that the degree of Bob's baldness is precisely 0.6, as opposed to 0.6001. Both would be good candidates for a first approximation. In reality, there is an infinity of first approximations (213). In a second phase, one should evaluate all possible first approximations; in a third level, one evaluates the truth values of the second approximations, and so on. Thus, a progression is triggered as soon as the truth value assigned in a first approximation is not taken as "definitive", as the only correct one for the sentence in question. In short, according to Smith, it cannot be that only one number is absolutely the correct truth value of a given sentence, all others being completely incorrect. Hence, the supposed need to have a new kind of truth value different from those in the interval [0, 1].

From here, Smith proposes that a sentence be assigned a degree function as its truth value. A *degree function*, DF, is a function from $[0, 1]^*$ to [0, 1], where $[0, 1]^*$ is the set of all sequences of a finite number of elements of [0, 1] (177). If f is a DF, it is represented by $<f_1, f_2, f_3, ...>$, each of the f_i standing for the real number in the unit interval being the truth value of the i^{th} approximation. For example, if the DF assigned to 'Bob is bald' is $<0.7, 0.9, ...>$, it means that it is true to degree ... that it is true to degree 0.9 that the sentence 'Bob is bald' is true to degree 0.7. So, the truth value, DF, assigned to a vague sentence is unique, but complex, i.e., it is internally structured by a hierarchy of truth values, each one picked up from [0, 1]. And the DF is considered as a degree of truth.

Smith says that, though ordinary speakers have access to the values of the first approximation (196), that is not the case for higher approximations, and this is the reason why higher order values are there. We do not have access to the full story about the degree of baldness of Bob. Smith claims that if we cannot have complete knowledge of any vague sentence's truth value, then, at the moment of assessing the validity of any argument, the only thing that matters is the value of the first approximation.

The second originality introduced by Smith is the notion of a *blurry set*, which is the denotation assigned to fuzzy predicates. A blurry set is a function relating individuals from the domain with a degree function.

We remark that beside the standard conjunction and disjunction, Smith's system contains only one negation, the weak one (188), in terms of which the conditional is defined, which we will symbolize by '$\supset$'. The truth value of "$p \supset q$" is exactly the same as "$\sim p \vee q$". The following table displays the truth values of Smith's conditional for a pentavalent logic.

$\supset$	1	¾	½	¼	0
1	1	¾	½	¼	0
¾	1	¾	½	¼	¼
½	1	¾	½	½	½
¼	1	¾	¾	¾	¾
0	1	1	1	1	1

5c.- Gradual Validity

Before judging the sorites paradox, we need to indicate the definition of *validity* employed. Smith seems to characterize a valid argument as the negation of an invalid one (192, 195). He holds that an argument is valid iff it is never the case that its premises are true enough to support a sound argument, while its conclusion is not true enough to be safely asserted. To be safely asserted, a sentence "p" need to be at least 50% true, because to assert "not p" will never be truer than to assert "p". And exactly this threshold is required for a sentence to be a tautology. As for the level of truth demanded for the premises to support a sound argument, their truth must be strictly greater than 0.5. So, a valid argument never has premises that are more than half true, while its conclusion is less than 0.5 true.

Summarizing, premises may have the property of being the basis for a sound argument, which requires from them a level of truth greater than 50%. The conclusion may have the property of being safely asserted, which means that it is at least 50% true. And a valid argument satisfies the condition that no assignment of truth values to premises and conclusion is such that the premises are more than 50% true whereas the conclusion is less than 50% true. This definition thus licenses the following possibilities.

Truth Level of:	Premises	Conclusion	Valid argument?
	support soundness	safely asserted	
	$> 50\%$	$\geq 50\%$	Yes
	$> 50\%$	$< 50\%$	No
	$\leq 50\%$	$\geq 50\%$	Yes
	$\leq 50\%$	$< 50\%$	Yes

5d.- The Sorites

Relatively to this definition of validity, the standard, conditional version of the sorites reasoning results valid but not sound, because many minor premises will have a value lower than 0.5 true. Furthermore, one instance of the major premise may be true to degree 0.5. In fact, checking the truth values -in the first approximation- of the premises, one observes that, while the atomic premises constantly diminish their degree of truth as the reasoning proceeds, the conditional premise is never totally true, bearing a truth degree fluctuating between very close to 1, near the extremes of the series, and 0.5, in the middle. If we had a series consisting of 101 members, then, two conditionals will have truth value 0.5, namely, $\sim p_{49} \supset \sim p_{50}$, and $\sim p_{50} \supset \sim p_{51}$. So, not all premises are true enough to form the basis of a sound reasoning, even though the argument is valid, for it complies with the negative requirement that there be no valuation assigning the premises a truth value greater than 0.5, and at the same time assigning the conclusion a truth value less than 0.5.

5e.- Assessment

Undoubtedly, Smith's definition of vagueness as Closeness is quite attractive, and conveys a basic trait of fuzziness. His repudiation of tolerance is also reasonable and should be enthusiastically endorsed. There is another undeniable virtue of Smith's theory, namely, its entirely keeping classical logic by means of a non classical semantics. However, the vocabulary of its language is not richer than that of CL, nor is its inferential power greater than in CL. Nonetheless, relative to other non classical or many-valued systems whose set of consequences is smaller than the classical one, Smith's theory has a privileged position.

On the negative side, there are at least six critical remarks that can be made. First, if we adopted degree functions as the truth degrees of fuzzy sentences, instead of the real numbers in the interval [0, 1], then it seems the logic dealing with them would be infinitely more complex than what is presumed by Smith. Indeed, since he assumes that there is not only one correct first approximation to the truth value of a fuzzy sentence but as many as there are real numbers in the unit interval, and given that there is one degree function for each first approximation, it seems to follow that there will be as many degree functions for each fuzzy sentence as there are real numbers in the unit interval. That is, a fuzzy sentence will have infinite degrees of truth. And this seems to complicate matters beyond the limits of what is convenient. And, if, in order to avoid the multiplication of degree functions, one demands that, at each stage of approximation, one should pick only that truth value located at the top or peak of the density distribution, then, this selection policy threatens to collapse the degree function into a single real number, which the theory was designed to avert. Moreover, this recommended strategy is practically useless for, as Smith admits, we are ignorant of the truth values given at any approximation of an order higher than the first.

Additionally, it seems to be unjustified to limit the number of entries in a DF to the finite case. Why should there be a stop to the unending process of "approximating" the degree of truth of a sentence? It appears that Smith is caught in an infinite regress problem. Notwithstanding, if it is really the case that «first approximations are accessible to ordinary speakers», as Smith acknowledges (196), then all subsequent approximations may, and should be avoided.

Second, it appears that, depending on how the initial circumstances are contrived, we can know -at least in some cases- the precise extent at which some property is possessed by an object, and, therefore, the truth value of the corresponding fuzzy sentence. I suppose this is the case for most bounded properties supervening on an underlying quantitative dimension, *G*, amenable to be measured. For example, the property of being far away from a given location, say Brussels. Bear in mind that there is a mark on Brussels' City Hall serving as -so to speak- the milestone number zero, a kind of reference point, from which to start the measurement. If we can agree on a point on the globe which is totally far from Brussels, that is, the farthest point on earth away from Brussels, then, once that is settled, how far from Brussels a point is is fixed, and we are able to calculate the distance. Perhaps not all cases are as easy as this one. But the example reveals the procedure we should follow to discover the degree of possession of a property and the degree of truth of the sentence stating it. If the objects form an ordered, bounded series, and the amount of difference in *G* between contiguous members is held constant throughout the sequence, then there should be no problem to discover the exact degrees involved. If there were difficulties, they would be due to our inability to finding out objective relations, but will not disclose any inherent indeterminacy in the situation, unless we adopt some sort of verificationism. It is only for the other cases, those of an unbounded series, that we do not know how to determine the exact degree of possession of the property in question due to our lacking a paradigmatic object serving as a standard of measuring. In these cases, we cannot be but agnostic about the truth value of the fuzzy sentences. See the next section § 6, for details.

In the third place, there is another important point that calls for a critical commentary. Smith affirms that the truth values in the unit interval do not obey the Tarski schema (2004: 210): «there is no reason why '*S* has truth value *x*' should have the same truth value as *S*». However, there seems to be an inexactitude here. The generalized Tarski schema does not correlate two degrees of truth, but a degree of truth with the extent to which something happens, or so it can be interpreted. Indeed, it seems natural to generalize the Tarski schema so that it comes to say that:

(GRT) '*a* is *F*' is true to degree Δ iff *a* is *F* to degree Δ.

We may call this thesis, the *Generalized Redundancy Truth*, though a strict equivalence should be in place of the mere biconditional. Smith himself (2004: 168, 183) recognizes that, at the abstract level, the (GRT) is a point of agreement between the many-valued and his own view. The difference between them lies in how the degree Δ is conceived: as a number in [0, 1], or as a degree function. Anyway, if (GRT) is right, and for some x in [0, 1], 'Bob is bald' is true to degree x, then Bob is bald to degree x, yet this second occurrence of 'x' does not represent a degree of truth, but Bob's degree of membership in the set of bald persons. When Smith contends that 'S is true to degree x' may not have the same truth value as S, he most likely is thinking about the first and second approximations to S's truth value. But their discrepancy is not a reason to dismiss the (GRT), for they are irrelevant, since (GRT) has to do with the semantic relation between the truth of a sentence and the fact it refers to.

Fourth, still another serious issue of concern is the definition of validity proposed (that no transition should be made from premises that are more than 0.5 true to a conclusion that is less that 0.5 true), for it has the startling consequence of authorizing as valid an inference having true premises to a degree less than 50% but a conclusion utterly false. This is just a special case of the last row of the alternatives enumerated in the table above, of section 5c. If all the truth of the premises is completely lost in the conclusion, then the reasoning does not preserve truth at all, and therefore, it must be classified, according to any standard, as invalid. Any definition going against this fact, seems to be simply wrong.

A fifth related source of worry comes from Smith's definition of the conditional, namely, /p⊃q/ is equivalent to /∼p∨q/, with weak negation in the left disjunct. Though this definition most probably keeps all classical tautologies, there is a conditional principle which is tautologous in *Aj* but loses its status according to Smith's definition of tautology. It is the *Acceptance or Endorsement Principle*:

Lp⊃p:
if a sentence is more or less true, then it is true, period.

But suppose that 'p' is only 0.49 true. Then, to affirm that 'p' is to some extent true is absolutely true. So, the degree of truth of the conditional is also 0.49, and so, it is not a tautology, i.e., it is not a formula being at least 50% true. The Endorsement Principle is like the touchstone of any sound gradualism. It gives us reason enough to be dissatisfied with Smith's definition of the conditional and/or his definition of a tautology (See Chapter 6, § 6d, for a defence of this principle).

Finally, it may be challenged that Closeness escapes contradictions, for if it is possible to link the extremes of the soritical series by pairs of objects which are very similar in *F* relevant respects, then it must be possible to pass from true simpliciter to false simpliciter trough the same means. But I am unable to see how this could be possible if there is no mixing or intermingling of truth and falsity. Yet, Smith accepts that opposites «blend into one another» (2001: 72). The problem for him is that he has not distinguished two sorts of negation. His logical system is such that it cannot tolerate a contradiction on pain of triviality, for it is not paraconsistent[17]. However, the system has the resources to be extended in that

17 A system is *paraconsistent* if it does not contain the Cornubia or Pseudo-Scotus Rule: p,
(continued...)

direction. Having degrees of truth, it can introduce different functors of affirmation and negation.

5f.- Smith's Defence of Degrees of Truth

Degree theoretic approaches have problems of their own due to their use of degrees of truth, which have been the target of several objections. Smith (2001, § 3.6) has provided a badly needed service to the fuzzy and many-valued community thanks to his defence of intermediate truth values. We now examine the details of the polemic.

i.- **On the very idea of a degree of truth**.- First, it has been objected that the talk of degrees of truth is just a mere façon de parler, a stylistic manner of talking about the truth of sentences.

Against this, Smith retorts that the degrees of truth are objects, concretely, the values of the functions that are the denotations of fuzzy predicates. And these functions are needed to account for the truth of sentences and the validity of arguments. Indeed, a semantic theory has to establish a relationship between the possession of a property, *F*, by an object, *a*, and the truth of the corresponding predication, '*Fa*'. In order to capture the intuition that the property can be possessed to intermediate degrees, functional values different from true simpliciter and false simpliciter need to be added. Thus, there must be as many degrees of truth as there are degrees to which a property can be instantiated (2005b: 178-9; 2001: 81). And these degrees of truth are objects. In an unpublished manuscript, Smith affirms:

> the further an object is in *F*-relevant respects from the paradigm *F* things, the less its degree of *F*-ness, the less the predicate '*F*' applies to it, the less true '*Fa*' is (where '*a*' denotes the object in question) (2003a: 11).

If Bob is bald to degree Δ, then the function being the denotation of the property baldness maps Bob into the value Δ, which is the truth value of the sentence 'Bob is bald'. Of course, Smith believes that, if Δ is a member of [0, 1], Δ should be taken as a mere approximation to the degree of Bob's baldness. But strictly speaking, Δ should be seen as a degree function, as indicated earlier. Anyway, degrees of truth are required elements in the model theory.

Again, some authors have complained that the notion of degrees of truth is obscure. For example, Delia Graff has condemned the lack of a substantial philosophical account of what degrees of truth are in themselves (Cfr. 2001b: 27).

Smith replies that a substantial account of what truth values inherently are is not needed. What is important is the structural properties of the algebra that they are a part of. An algebra of the type we are interested in to model vague predicates comprises a set of truth values plus a set of operations defined on them. If we think that the predicates accord with bivalence, then the algebra is Boolean, that is, the set of truth values is {0, 1}, and the operations on them are $\wedge$, $\vee$, and $\neg$. Whereas, if we believe that the predicates admit of gradations, then the structure is a different algebra, having distinct functions operating on a different set of truth degrees. For Frege, the truth values, the True and the False, were definite objects, while for structuralists there is no unique set of truth values that can play the role. In either picture, bivalent or not, what matters is not what the truth values are like in themselves but the properties of the structure.

However, personally, I think that Graff's demand is justified. And a plausible answer is that we need to look at the ontological side of a degree of truth. The function being the

(...continued)
not p $\vdash$ q. Because this rule is invalid in such a system, it can contain a pair of contradictory formulas without trivialization.

ontic correlate (the denotation) of the predicate 'bald' assigns Bob the degree of his being bald, and this is the fact that he is bald. This fact, that Bob is bald, is nothing but Bob's being bald, i.e., the baldness of Bob, and, if this particular property is a matter of degree, so too that fact is a gradual matter. On this issue, Williamson (2003: 700) is right in countenancing the thesis that state of affairs are truth values, for these are the elements correlated with sentences by the semantics. Smith perhaps will not reject the idea that the best foundation for degrees of truth is that existence itself is a matter of degree. In [2005a], he defends that there is a sense in which existence is not an all or nothing matter; in effect, objects may possess the property of existing at world w to intermediate degrees.

Finally, as another version of this worry about the nature of the truth degrees, it is said that some fuzzy or many-valued theorists confuse degrees of possession of a property with degrees of truth. Rosanna Keefe (2000: 91-4) argues that it is wrong to reason in the following manner: since *a* is taller than *b*, then *a* has the property tallness to a greater degree than *b*, and that, therefore, *a* satisfies the predicate 'is tall' to a greater extent than *b*, and so '*a* is tall' must be truer than '*b* is tall'. According to Keefe, the inference can go through only if one illegitimately switches from one sense of 'degree' to the other. Graeme Forbes is singled out as having committed this mistake.

Smith agrees that these two sorts of degrees, of possessing a property and of truth, should indeed be set apart. It should not be a component of the fuzzy view that, if *a* is taller than *b*, then '*a* is tall' is true to a greater extent than '*b* is tall'. What Smith accepts is that, if *a* is taller than *b*, then the degree of truth of the sentence '*a* is tall' is at least as great as that of '*b* is tall', that is, /*a* is tall/ ≥ /*b* is tall/. In order to realize this, Smith invites us to consider the heights of Kareem Abdul Jabbar, and of Larry Bird, who are 2.15 m, and 2.02 m high respectively. Although Kareem is taller than Larry, 'Kareem is tall' is not true to a greater degree than 'Larry is tall', for both sentences are true simpliciter. Hence, the degree to which a property *F* can be instantiated by an object *a* is distinct from the degree of truth of the statement '*Fa*'. However, despite the distinction among these two kinds of degrees, Smith claims that the transit from the one to the other is authorized by Closeness.

We disagree with both, Graff and Smith on this subject, and side instead with Forbes. There seems to be a confusion of semantics and pragmatics here. On this particular, see my discussion of Williamson's arguments against the many-valued view, in the last section of this chapter: § 7, and § 1e of the Introduction.

ii.- **The Correct Interpretation**.- Another objection against the gradualist view is that one cannot see what could possibly fix that the extent to which Bob is bald is 0.654 as opposed to 0.653. And if we cannot find a way to determine the exact degree, then there is no fact that will help us in deciding the question.

The answer Smith offers in this connection is that the degree theorist is here not worse than others, since the problem of how the exact interpretation is fixed challenges all extant theories of vagueness. So, nobody is justified in alleging that whatever fixes the reference of vague words does not favour the fuzzy view. Nor can it be concluded from the fact that we have no idea about how to discover the particular degree of Bob's baldness that there is no unique degree of Bob's baldness. In this respect, the degree theorist adopts a line of defence that resembles that of the agnosticist. Remember that Williamson argued that, it is not because we ignore the way in which use and environment factors draw the sharp boundary to the meaning of vague predicates, that we are entitled to infer that there is no such clear-cut dividing line. Until we have a better understanding of the mechanisms of meaning fixation, we are in no position to discard the gradual approach on the grounds that it has not provided a plausible picture of how meaning is determined, for other approaches are on the same footing.

iii.- **The Last Bald Man**.- A third objection charges the degree theorist with positing a boundary as sharp as in a classicist theory. In fact, considering the soritical series having as opposite extremes bald and hairy men, there will be a last man, a_i, who is bald to degree 1, and his next neighbour, a_{i+1}, will be bald to degree less than 1. And, we can add that, if the only designated value is 1, then this boundary seems to be no better than the one afforded by the classical picture, for which 'Fa_i' is true simpliciter, and 'Fa_{i+1}' is false simpliciter.

Smith retorts that the difference between 'Fa_i' being true to degree 1 and 'Fa_{i+1}' being 0.999 true, in as long as both truth values are not the same, constitutes a sharp boundary, which, nonetheless, should not be censured for it does not violate Closeness: indeed, both sentences are very close in respect of truth. Given that the sentences are both true -if indeed this is the case-, the assignment of distinct truth values to contiguous sentences in the series respects the vagueness of the predicate.

iv.- **Truth-functionality**.- A last problem that need be addressed has to do with the fuzzy negation and the ensuant contradictions that the fuzzy view tolerates. If a borderline sentence, "p", is half true, its negation, "∼p", will also be half true, and, consequently their conjunction, "p∧∼p", will result half true too. This valuation has appeared objectionable to some philosophers. Thus, Williamson (1994b: 136) maintains that:

> 'He is awake and he is asleep' has no chance at all of being true... the conjunction in question is clearly incorrect. ...Intuitions can be confused by the idiomatic use of contradictions such us 'He is and he isn't' to describe borderline cases.

In reply, Smith says that the fuzzy picture can accommodate the belief that a contradiction will never be clearly true, for it can be at most 50 % true. But from the fact that a contradiction is not clearly true, it does not follow that it has to be clearly false, for the gap can only be bridged if we make some dubious assumption. On the contrary, people do describe a borderline case contradictorily. For example, they would say that a point midway between clearly red and clearly orange is sort of red and orange, a bit of both (86). In fact,

> we do not think that it is definitely false to say of an object somewhere between red and orange that it is red and orange... (71; 88, n).

And the reason for this is that:

> if it is somewhere between red and orange, then it is to some extent red and to some extent orange (71).
>
> The typical feeling about bald and hirsute, short and tall, red and orange, and so on, is that these properties *blend into one another* (87).

Thus, when we are confronted with borderline cases, it is not incorrect to assert contradictions. From this perspective, it seems that Williamson has not explained why the parlance of people is confused. His unwillingness to take literally the contradictorial usage seems to be an example of a philosopher who has decided to dismiss some particularly troubling usage, to which he is antecedently committed.

On the other hand, it seems Smith suffers a tension between accepting contradictions for philosophical reasons having to do with fuzziness, and prohibiting them, by force of his logical apparatus. He would be better off if he modified his system to make room for his contradictorial intuitions.

6.- On the Measure of Degrees of Truth and of Membership

Terence Horgan (1994a: 161-2) has criticized many valued logics on the ground that they introduce arbitrary precision in the realm of vagueness. His argument is as follows. The norms of proper usage do not favour any assignment of a precise degree of truth to a predication, over against other plausible candidate assignments. Furthermore, for any vague predicate, there is a variety of functions, linear and non linear, that could be «equally consistent» with the ordinary usage of the predicate. But which function among the several alternatives is the correlate of the predicate, and why? There is no principled answer to these questions, and this fact, which is crucial for vagueness, reveals that any choice in this respect would be arbitrary. Hence, the reasoning may continue, either there is a mysterious correlation between the predicate and its ontic counterpart, or there is no fact of the matter as to which specific function is picked up by the predicate. And this mistrust is generalized in the philosophical research community. There is a widespread skepticism about the meaningfulness of degrees of truth (Hähnle: 314), and a suspicion of «the extremely artificial nature of the attaching of precise numerical values to sentences» (Urquhart: 286, 291).

Horgan's objection is so deep and serious that even many gradualist thinkers are convinced by it. Thus, some fuzzy and many-valued logicians have acknowledged that the precise degrees of truth should not be taken too seriously (Lakoff: 462, 481); that particular numerical values are not much meaningful (Goguen: 332); that what really matters in the set of truth values is the order among them (Machina 1976: 61). Likewise, Dorothy Edgington recognizes the arbitrariness of any assignment of a precise degree of truth to a sentence, given that no exact numerical value is uniquely correct (2001: 372, 375); a precise account of vagueness is then unrealistic (1996: 308). Gradualist Cook adopts an instrumentalist perspective of the degrees of truth. For him, although truth comes in degrees, the specific real number assigned to a sentence is merely a convenient tool serving the function of simplifying the theory. So, the actual degree assigned should not be taken as representing reality, but as a simple artifact (2002: 238-40). Bilgiç and Turkşen, in their general survey on the measurement of the degrees of membership to fuzzy sets, conclude that the only meaningful approach to membership functions is ordinal (218). Dubois and Prade (1997: 142), after giving an overview of the most prominent conceptions of the meaning of the degrees of membership and the methods of their measurement, gather that these questions of the meaning and measurement of degrees remain partially unresolved. The last two authors, together with Ostasiewicz, have also accepted that it is not always possible to numerically represent the degree of membership of an object to a set (Dubois, Ostasiewicz, Prade: 28). Even Philippe Smets and Paul Magrez [1988], who try to vindicate the meaningfulness of a precise evaluation of the degree of truth and the degree of membership of an object to a set, draw the conclusion that the determination of the value at which /John is tall/ = /John is not tall/ is personal, corresponding to a personal interpretation of the word 'tall'. «One should not hope for an absolute, supra-human, individual-independent meaning of the word» (71).

Recapitulating, many gradualists are persuaded that a quantitative approach to fuzziness is flawed, trying instead several alternatives. One is to replace the cardinal approach by an ordinal one. A second one is to take the talk of degrees as a useful fiction in the modelling of fuzziness, not corresponding to anything real (instrumentalism). A third is to admit that the degrees attributed are subjective. A fourth alternative construes the set of degrees of truth as a probabilistic structure. A fifth is to take the exact values as mere approximations. And so on.

There are other answers to the problem. Here is our solution. The prototype interpretation of the degrees of possessing a property is an attractive model. According to it, the extent to which an object *a* possesses a property *F* is determined by its similarity to a paradigmatic exemplar of *F*. The exact degree to which *a* is *F* is calculated by *a*'s distance to the prototype of *F*. However, the scope of application of this method is limited, since it is restricted to those soritical series that satisfy the following conditions: (a) that there are effectively two objects that exemplify, respectively, the property and its opposite to a total degree; (b) that we know how many members the series may contain; and (c) that the amount of difference in the underlying dimension, *G*, between any two contiguous elements of the sequence is kept invariably fixed. For the other cases, for example, when the soritical series is open on one side, this prototype procedure is helpless.

We admit that there are a number of possible functions that could satisfactorily model a given fuzzy predicate, and correspondingly, there are several admissible assignments of an exact truth value to a fuzzy sentence. Which function and assignment are the uniquely correct? There is one, but we do not know which, and we do not need to know either. Mother Nature has provided us with knowledge of things that are adequate to our vital needs. We do not need to know the exact extent to which a flea is a large flea, a large animal, or a large thing. We are not omniscient.

Furthermore, with a predicate like 'old', as applied to humans, we do not know how to measure the degree of oldness. For one thing, the oldness of a person is not a function of only her age; what other factors contribute to her oldness is a much more complex affair, and rather unknown at the present stage of investigation; it may be that we are just beginning to understand the process of aging. Second, life expectancy of people is being increased in the last decades. So, the correlation between the predicate 'old' and its corresponding membership function varies with time, with the historic evolution of man. Third, perhaps some day in the future it can be demonstrated that nobody can live more than, say, 500 years. Thus, there is a possibility that we may come to know the existence of a limiting case, a point of reference with respect to which the measure of similarity can be done.

On the other hand, it seems that Horgan is employing a *gnosticist* presupposition in his argument, to wit, that what is determined in reality is determined epistemically, in the sense that we either know that the case is true, or we know what things would be required to know in order for us to know that it is true. Indeed, he is taking for granted that, if there is a unique correlation between a fuzzy predicate and its membership function, then we should be in a position to know it; but, since we do not know that, then it is ontically indeterminate which membership function there corresponds to the fuzzy predicate. However, that gnostic assumption is too sanguine. And we could call '*agnosticism*' the position that denies this epistemic optimism: there are facts beyond our ken. That there is actually, in reality, only one function that corresponds to the fuzzy predicate is not arbitrary. Yet, the ignorance we profess is not incognoscibility in principle, unknowability for any intelligent mind, but just human beings.

7.- On Williamson's Objections Against Many-Valued Logics

In order to make his agnosticist view plausible, Timothy Williamson raises several objections against alternative approaches in general, and many-valued systems in particular. Here we undertake the necessary task of replying to several objections he has contrived against the degree-theory. Thus, we do not ignore the difficulties our perspective faces, but take issue with him. (Cfr. Williamson 2002c: 52).

In a three valued logic incorporating a weak negation, '$\sim$', and a truth-functional conjunction '$\wedge$' (whose truth values are determined respectively by: $/{\sim}p/ = 1\text{-}/p/$, $/p\wedge q/ = \min\{/p/, /q/\}$), when 'p' is a half true sentence, its weak negation receives the same value too, and also their conjunction: $/p/ = /{\sim}p/ = /p\wedge{\sim}p/ = ½$. Commenting on this, Williamson says that it is absurd. And he takes this as a reason for abandoning the truth-functionality of conjunction. «How can an explicit contradiction be true to any degree other than 0?» (1994b: 136-137). However, this remark begs the question against many-valued paraconsistent approaches, presupposing what should be argued.

Another criticism Williamson levels against many-valued approaches is that a many-valued semantics invalidates classical logic, in that the Principle of Excluded Middle, "p or not p", no longer holds (*Ibid*.: 128-129; 2003: 695). According to that non classical semantics, there are contexts in which neither "p" nor "not p" are «clearly» true; that is, standard cases of fuzziness -understood agnostically as obstacles to knowledge- would be counterexamples to PEM.

But this worry does not apply to all many-valued systems, although it is true of Łukasiewicz's logic, due to the Polish Logician's policy of designating just the maximal value 1, leaving all intermediate values without any alethic status; for him only perfectly true sentences were true (*maximalism*).

We believe that such a requirement is excessive, and instead adopt the opposite stance: all values greater than 0 are designated (true); and symmetrically, all values under 1 are antidesignated (false).

Thus, within our approach the PEM and the principle of non contradiction are retained, as well as all other classical tautologies as long as we understand the classical negation as a strong one. (We understand a *tautology* as any formula which takes a designated value independently of the truth values of its subformulas.) But of course, many other truths are added.

Furthermore, we should distinguish between epistemic indecision and objective indeterminacy; only the latter would challenge the PEM, not the former, unless we were verificationists. But, we must not mistake uncertainty for lack of real determination. Since we do not accept objective indetermination, there is no reason against PEM.

Williamson points out another source of misgiving. Suppose that *a* and *b* are, respectively, the tallest and the second tallest persons in the world now living. In his opinion, what is troublesome is that '*b* is tall' would not be perfectly true -according to the lights of many-valued approaches- although it is quite straightforwardly true [1994b: 126-127].

Notice that what prompts the suspicion is not the comparative notion of degrees of truth, *truer than*, which is analysed in terms of the degrees of possessing a property. Since *a* is taller than *b*, then '*a* is tall' is truer than '*b* is tall'. This is granted by Williamson. The problem is that, supposing *b* measures more than, let's say, 3 m, *b* is extremely tall, and therefore, for all practical purposes, '*b* is tall' should be assigned value 1. In other words, Williamson suggests that the few first members of the sequence of men ordered by the relation *being taller than* should all be given the maximal value.

But if this were so, the general identity between degrees of truth and degrees of possessing a property (GRT) would fail, which would undermine all the foundation of fuzzy or many valued logics. On the contrary, all differences, even the smallest ones, must be taken into account. Person *b* may be very very tall, but not absolutely tall, for he is shorter than *a*. We cannot do away with minimal differences. Even if for all practical purposes, one thousand (or several thousand) men are *as if* -so to speak- they were completely tall, they are in fact not completely tall, except the first one, were they ordered in a series. In a particular context, we can adopt a very high truth threshold -in effect disregarding alethic differences beyond that threshold. But such a policy is pragmatic, a matter of communicational convenience alone. Lorenzo Peña says:

> Williamson's example of the two or three tallest men assumes that they are unusually and extremely tall; hence our reluctance to acknowledge that all of them, except the tallest one, are partly non-tall. Under unusual circumstances our spontaneous reactions are unusually constrained and indeed stretched. Hence the quandary and the qualm Williamson detects. We had better consider more run-of-the-mill cases, like the one of gradual transition from purely red to purely blue ink by a progressive dose
> (Peña and Vásconez, Unpublished).

Paraphrasing Williamson (1997b: 923), we conclude that, if asked for the alethic status of a fuzzy sentence, instead of answering: «it is not clear», one should add more truth values to the classical ones.

On the other hand, Williamson refuses to take a paraconsistent approach seriously. He affirms:

> ...no attempt will be made to argue with those who think it acceptable to contradict oneself (1994b: 189).

All contradictions are absurd (1992: 147, n. 4). Thus, efforts like those of Sylvan and Hyde (1993: 647, 649), Machina (1976: 59), and others who use systems tolerating contradictions are dismissed beforehand, without any examination of their virtues and defects. But Peña retorts:

> Generally contradictions are spurned as self-stultifying on account of their purported blocking or paralysing effect of leaving us with no reasonable way of finding out what is at stake; or else on account of the opposite effect of licensing any conclusions (via the Cornubia rule). The former charge means that, once you have committed yourself to utter "p and not p", you are thereby cornered into such a quandary that any further inquiry would become pointless or foreclosed. The latter charge allows you to glibly avail yourself of the Cornubia rule in order to reach any far-fetched conclusion you may happen to want. But in effect the result is the same: once you have contradicted yourself, anything you go on to say is useless and unenlightening.
>
> While our gradualistic approach entails many contradictions, each of them can be explained, elucidated, assessed on its own. The inquiry is not blocked by any of them. No such contradiction is the end of the story. None of them is fully true, either, since we accept the principle of non-contradiction and claim that all contradictions are at least partly false.
>
> No contradiction merely happens to be true, so to speak by luck or ill-luck (as a sheer result of a fact and its negation being both present in the world as they could have been absent -everything else remaining unchanged; in such a case we would have a brute self-contradiction). On the contrary, within our paraconsistent approach we analyze the nature of contradictory truths as the emergence of mixtures, as something which supervenes on degrees, each degree being a moment in a transition of a blending process. Thus Williamson's flippant rejection of any paraconsistent approach is not justified (Peña and Vásconez, Unpublished).

CHAPTER 5

OTHER APPROACHES

After examining agnosticist, supervaluationist, indeterminist, and many-valued theories, we have seen the major proposals in our research field. However, there remain other currents, less popular, but active in the debate. We now take a look at them. They are: intuitionism, nihilism, paraconsistent approaches, and contextualism.

1.- Wright's Intuitionism

Crispin Wright is an author who has been publishing on our two topics, vagueness and the sorites paradox, for more than 3 decades, since 1975. Over the years, he has changed his convictions. We are going to trace the origins of his position, but we will concentrate on his current stand. The main thrust of his thought is intuitionist, though it also contains elements of a broadly epistemic indeterminism.

1a.- The Beginnings and Maturation of the Reflection

The two articles [1975] and [1976] contain more or less the same ideas, and we treat them as the first phase of Wright's intellectual evolution. Then we will refer to his [1987a], which marks a major step forward, and allude to his [1994] in passing.

The problem as originally perceived is that the meaning of vague expressions seems to prohibit sharp boundaries. That is, the ancient persuasion that a heap does not transform itself into a non heap by losing just one grain appears to be true. A small change will not make a difference. Thus, $\neg\exists a_i\ (Fa_i \wedge \neg Fa_{i+1})$, the classical version of the *continuation principle*, i.e., one formulation of the major premise of the sorites, would seem to be part of the meaning of vague predicates. Vague terms would have blurry boundaries. Nonetheless, this principle $\neg\exists a_i\ (Fa_i \wedge \neg Fa_{i+1})$ leads straight to paradox, and «*incoherence*»: the application of the predicate '*F*' would be extended to cases where it is not applied. And something analogous occurs with observational words, like those for colours. It seems obvious that, if two objects are indistinguishable, then both merit the same verdict. This is a variant of the *similarity principle*. But in a soritical series, this second principle brings about disaster.

Though one may think that there is enough motivation behind those two principles, of continuation and of similarity, Wright remains unconvinced, and, from the beginning, he is suspicious of them. He imposes himself the future task of looking for a basis for describing differently the indiscriminable members of a soritical series (1975: 352). It seems that there is no other escape than to find some specific fault with the major premise of the sorites (1987a: 244). This means that the project Wright is going to embark on is the one of reconciling vagueness with precise boundaries:

> ... there need... be no substantial sacrifice in endowing formerly observational predicates with exact boundaries... (1976: 243);
>
> observationality... is consistent with the existence of a sharp boundary (1987a: 246).

Another ingredient of his early treatment of the riddle of vagueness is his acknowledgement that there is something correct about bivalence: that, presented with an object, we have only two options: either to apply or to withhold the predicate in question (1975: 350).

Let us see now some core developments of these basic tenets, in his [1987a]. There is a clear perception that $\neg\exists a_i\ (Fa_i \wedge \neg Fa_{i+1})$ poses a dilemma. If it is true, then there is no transition in the soritical series from things that are *F* to things that are not *F*, and all of them will be *F*. On the other hand, if the principle is false, then we are saddled with a determinate sharp boundary that strikes us as fictitious. Wright chooses to wrestle with the second horn of this dilemma, entertaining the possibility of considering the major premise false. But, concerning the difficulty introduced by the first horn, he does not feel obliged to provide an explanation to the question of how the transition is done from one opposite to the other, dismissing it as spurious (1987a: 268).

I conjecture that, for Wright, there is only a little truth in $\neg\exists a_i\,(Fa_i \wedge \neg Fa_{i+1})$, for the removal of a grain from a heap cannot transform it into a non heap. But, this does not mean that the continuous removal of grains will never cause a change (1976: 230). Again,

> a subject who was prepared to describe one but not the other of a pair of indiscriminable color patches as 'red' would invariably give cause to think that he misunderstood the predicate. ... But it is not obviously so precisely in the context... of a Sorites series: here we should want to recognize the right of the subject to 'switch off' at some point... (1987a: 269).

So, indiscriminable objects should be treated equally, except in the context of the sorites. In view of the paradox, we are not allowed to treat indiscriminable objects equally. The small attraction of the major premise does not make Wright lose faith in the exclusivity of the predicate, i.e., in that it is not applied everywhere. So, the major premise of the sorites is now considered to have been reduced to the absurdum. It is untrue; or more precisely, it is not definitely true. What the paradox shows is that the major premise cannot have a determinate truth value (1987a: 267). However, Wright is still reluctant to consider, in general, the major premise as false. He wants to block the inference from "not definitely p" to "not p" (1994: 142-44). On the other hand, he is trying to overcome the scruples that there is a last member of the soritical series that is definitely *F*, or a first one that is not definitely *F*, though they may be difficult or impossible to be identified (1987a: 245, 256). The task now is to find why $\neg\exists a_i\,(Fa_i \wedge \neg Fa_{i+1})$ cannot genuinely express what vagueness consists in (1987a: 287, n. 35).

With a view to discredit the major premise, Wright attacks one of its possible grounds. Some philosophers have vindicated the concept of an *observational predicate* as one which satisfies (a variant of) the *similarity principle*, that, if two objects are indistinguishable, then a differential judgement about them cannot be warranted: a predicate applies to both if to either. We note that Wright has vacillated over the present issue. In his later criticism to Williamson, Wright (1994: 151), with good reason, agrees with the mentioned definition, complaining, against Williamson, that:

> it is... absurd to... justify incompatible color judgements about items that look exactly alike.

Notwithstanding, in (1987a: 247), Wright thinks that the given characterization of the observationality of predicates is a misunderstanding. He wants to maintain the opposite opinion that only one of a pair of indistinguishable members of the series is definitely *F*. If it were true that all observational terms comply with the major premise of the sorites, then all of them would be incoherent, there would be no observational vocabulary, and the connection between language and empirical reality would be put in jeopardy. Wright, then, has to provide an alternative conception of what observational terms are.

He believes that a partial source of the paradox is the notion of an objective meaning. Correspondingly, he defends that:

> sense can be given to an expression only by reference to conditions whose satisfaction we can determine at least in principle (1975: 357).

> We are ceaselessly actively involved in the determination of meaning (1987a: 281).

Consequently, Wright proposes that an object x has an *observational property*, *F*, if competent speakers assent to '*Fx*' (1987a: 276). An object *definitely* has the property *F* if there is a consensus among the speakers. Thus, to be red is to look red. If observational predicates are dependent upon the reaction of subjects, then we have a way to understand that in the soritical series there is a last element that is definitely *F*, for, if a group of speakers is successively asked of each object in the series whether it is *F* or not, there must be a point where the consensus is broken.

1b.- The Quandary and Permissibility Views of Borderline Cases

We now proceed to give the details of Wright's present perspective, as developed in his [2001] and [2003c] articles. (Unless otherwise noticed, references in this subsection are to his paper of 2001). In order to clarify what is most crucial to vagueness, it is useful to contrast Wright's conception with that of bivalent agnosticism, as supported by Timothy Williamson, and with indeterminism properly so called.

First, Wright contends that vagueness is an "epistemic" notion. A borderline case, *a*, of application of a vague expression, '*F*', is to be characterized in terms of ignorance. So, when a subject is presented with a borderline case, *a*, and is asked whether it is *F* or not, she is baffled, uncertain, and does «not know what to say» (77). A *borderline case* consists in the inability of a competent judge to come to a polar verdict (2003b: 471). A theory taking this agnostic general direction is correct. However, Wright's view of vagueness differs from Williamson's agnosticism in that the former does not include undecidability. It is «essential» (81, n. 41) that one who is in a quandary falls short of knowing that it is impossible to know whether *a* is *F*. A claim of unknowability should not be advanced.

Thus, vagueness, and borderline cases in particular, are a special type of quandaries. More technically, the key suggestion is that "p" is a *quandary* for a competent subject, who is in good conditions for judging the situation, if the following four requirements are met: (i) she does not know whether "p" is the case or not, (ii) she does not know any way of ascertaining whether "p", (iii) she does not know whether there is any way of ascertaining "p", and (iv) she does not know whether there could be any way of ascertaining "p".

Before continuing, it is convenient to elucidate the conditions under which a statement is unknown. For reasons having to do with indeterminism, Wright rejects the following thesis:

(AG) "p" is *unknown* if it is possible that there is a "q" entailing the negation of "p", and one is in no position to exclude "q".

In more simple terms, what (AG) says is that one does not know what might be false. We are justified in denying a knowledge claim to "p" if one is not able to exclude all possibilities that "p" is false. Wright argues that, even if we know that there is no true "q" inconsistent with "p", that is not enough to know "p", precisely because there is a range of predications in which we are in a quandary. This is connected with the intuitionist refusal of the principle of double negation elimination: "$\neg\neg p \supset p$". What else is needed to endorse "p" beyond the knowledge that "not p" cannot possibly be true, is a positive guarantee of the truth of "p". Thus, instead of (AG), Wright proposes:

(AG^{+}) "p" is known if there is assurance that evidence in favour of "p" will be feasibly acquired.

By contraposing (AG^+), one obtains that "p" is unknown if we are not certain about getting evidence for "p". This characterization of what does it take for "p" to be unknown is more in line with the intuitionist doctrine that there is truth only when there is a proof of it.

Second, if 'p' is a sentence describing a borderline situation, then its vagueness does not consist in its incompatibility with both polar verdicts (that it is true, or that it is false), lacking any alethic status whatsoever. p is not indeterminate in the sense of being inconsistent with either the positive or the negative verdict. p is not a truth value gap, nor does it receive a third non standard truth value. Rather, either of the polar verdicts is permissible. That is, normal subjects making opposite informed descriptions of a borderline case under optimal circumstances for judging are all entitled to their «suitably qualified» conflicting opinions (2003c: 94). Presented with a borderline case, a person may utter a polar verdict, either positive or negative, without revealing any lack of competence in the use of language.

The source of vagueness lies in the possibility of doxastic disagreement, and we are in no position to say that one of the disputants has made a cognitive mistake.

The permissibility of divergent inclinations is due to there being no fact of the matter that proves one of them incorrect. No fact will make one of the judgements defective or disclose some intellectual shortcoming in the process by which they were arrived at. If there were an objective fact, then people could reach a consensus about the subject matter of discussion; but, we have no idea about how to settle the dispute (70), and it is highly probable that there will be no consensus reached; therefore, there is no state of affairs that could prove one the disputants wrong, and the other right. Hence, one may endorse an opinion but without warrant (2003c: 93). When there is difference of opinions concerning a borderline case, there need be nothing to choose (56). And yet, utterances about borderline cases are truth apt, or truth evaluable, and about a knowable matter (59, n. 16). Thus, Wright is against the existence of a truth making state of affairs (58-9). Therefore, he claims that «the absolutely basic datum» (70) concerning vagueness is that we cannot make a knowledgeable or warranted decision as to how to classify the borderline object.

So, according to Wright, a factualist position, holding that there cannot be a disagreement of opinions without a mistake being attributed to one of the parties, is in error. Indeed, factualists support the realist idea that the world is thus and so independently of us and our practices. Wright contests this attitude. In discourses dealing with matters of inclination, as intuitively opposed to discourses responding to how things stand, those «matters are... constitutively dependent upon *us*» (59. Cfr. 1987a: 244-45).

Consequent upon this anti-realist viewpoint, there is a third, minor feature of vagueness. «All» common examples of vague expressions, like 'bald', 'heap', 'red', 'tall', 'child', etc., are subject to the *Evidential Constrain* (80, n. 39; 95), namely:

(EC) If "p", then it is feasible to know "p".

And the negation of "p" may perfectly be an instance of substitution of "p" in this schema. Collor predicates illustrate this characteristic very well. Because the colour of a surface is a matter of its visual appearance, if x is red, then it appears as red. This is why colours are said to be *transparent*. But in contexts where (EC) is enforced, the Law of Excluded Middle would imply that, for all "p", either it is feasible to know that "p", or it is feasible to know that not "p". Yet it is uncontroversial that, for some propositions, we currently lack a procedure of decidability, so that it is not the case that, for all propositions, either it is feasible to know that p, or it is feasible to know that not p. Therefore, the LEM is not warranted in the presence of (EC), and sorites-prone predicates are evidentially constrained. But note that the LEM is not false, but only not justified. And the same conclusion follows concerning the Principle of Bivalence (2003c: 101). In areas where (EC) is applied, the PB says that either p is knowably -so, definitely- true or false (91). Thus, in general, where there are quandaries, or (EC) applies, classical logic should be suspended.

1c.- The Intuitionist Solution to the Sorites

Wright reasserts that the sorites is a reductio of the major premise. The paradox demonstrates that it is false that there is no a_i such that a_i is F but a_{i+1} is not F. In logical notation, the sorites shows that: $\neg\neg\exists a_i\,(Fa_i \wedge \neg Fa_{i+1})$. However, from the negation of the major premise, it does not necessarily follow that there is an a_i such that only a_i is F but not a_{i+1}. Wright calls this latter thesis the *unpalatable existential*, which we will symbolize as:

(UE) $$\exists a_i\,(Fa_i \wedge \neg Fa_{i+1})$$

The (UE) is not a logical consequence of $\neg\neg\exists a_i\,(Fa_i \wedge \neg Fa_{i+1})$, for the double negation elimination fails. Thus, Wright believes that the denial of the major premise is made compatible with vagueness. Vagueness does not support that $\neg\exists a_i\,(Fa_i \wedge \neg Fa_{i+1})$ (2003c: 99). That premise can be denied without problem because vagueness is an epistemic phenomenon (84). And the same solution applies to the universally quantified conditional version of the premise, $\forall a_i\,(Fa_i \supset Fa_{i+1})$. The sorites proves its falsity; but, for the same reason, "$\neg\forall$xp" does not entail "$\exists$x$\neg$p": double negation elimination does not licenses this inference. Furthermore, the *similarity principle*, that a predicate applies to both or none of a pair of adjacent members in a soritical series, should also be resisted (80). Wright thus reiterates his rejection of the sound proposition that a divergent judgement about contiguous elements in the sequence cannot be justified, no matter how close they lie. Therefore, Wright wants to dissociate vagueness from the major premise of the sorites. Nor should vague predicates necessarily be connected with sorites susceptibility (1989: 130-31).

Wright admits that the (UE) expresses what it means for a word to be precise. Despite his non commitment to the truth of (UE), he does not exclude the possibility that it may be known to be true (83, and n. 44). But, even if it were true, there should be no reason to be worried. (UE) cannot pose a threat to vagueness, since

> it was a mistake to view vagueness as entailing a *lack of precision* (83).

Vagueness does not contradict (UE). Hence, Wright's opposition to the following conditional:

If $\exists a_i\,(Fa_i \wedge \neg Fa_{i+1})$, then '*F*' is not vague.

This is unacceptable for him. Since its antecedent may be true, but its consequent is false («for *F* is vague by hypothesis» *Ibid*.), the whole conditional may also be false. It is misconceived.

Indeed, (UE) also presents a quandary.

1d.- Closing Commentary

The reader may surmise how far my own position is from that of Wright. I do not share his anti-realist view of observational predicates, nor his response-dependent conception of discourses about matters of inclination, nor his abandonment of the truth maker principle. (For my reasons against Wright's stance on these points, I refer the reader to Chapter 1, § 1e). His dismissal of double negation elimination, and the equivalence between "$\neg\forall$xp" and "$\exists$x$\neg$p" is also to be regretted. Furthermore, his refusing to provide an explanation to the legitimate question of how the transition is done from one opposite to the other in the soritical series is unsatisfactory. And his *maximalism*,

> that truth is absolute -there is, strictly, no such thing as a proposition's being more or less true; propositions are completely true if true at all (2001: 54, n. 10),

is all the most unwanted. (For a discussion of maximalism, see Chapter 6, § 6d). The loss of the major premise of the sorites is something totally contrary to fuzziness. I maintain that fuzziness does provide a foundation for the truth of the major premise. On this matter, I earlier remarked that Wright has hesitated. I subscribe the position he takes in his criticism of Williamson's agnosticism. There, Wright cogently maintains that:

> our standards of justification find no significant distinction between the adjacent elements in sorites series (1994: 151).

A point that I would like to raise for discussion is his doctrine of borderline cases as permissibility of both polar verdicts. Let me quote one of his characterizations:

> for an item to be a borderline case on the red-orange border is for it to have a status consistent both with being red and with being orange, (so not red), precisely because... it *has not been determined* whether it is red or not (1994: 139).

It would be against the intentions of the author to interpret this passage as a hint of Wright's acquiescence to allow that the borderline case is both red and not red. However, in his earlier writings there are also signs displaying a more open attitude. Here is a quotation.

> It is ... unclear how far our use of e.g. the vocabulary of colours *is* consistent. ...it would be unwise to lean too heavily, as though it were a matter of hard fact, upon the consistency of our employment of collor predicates (1975: 361).

Really, it should be reckoned that to utter a contradiction is the best way to describe a borderline situation. If this is so, then a more liberal disposition towards authentic borderline cases might concede the possibility of their being contradictory.

2.- Nihilism

Generally speaking, *nihilism* is the position that denies the existence of vague properties or objects. Some representatives go further and affirm the meaningless character of vague expressions. We introduce in this section the views of two prominent theorists of this trend: Peter Unger and Terence Horgan.

2a.- Unger

Peter Unger [1979] has developed an incoherentist conception of fuzziness, and a nihilist response to the sorites. (Since the references in this section are all to the same publication year, 1979, we cite his articles by just mentioning the distinguishing letter: a, b, or c). But, Unger abandoned his radical nihilist position at least since 1990.

Concerning vagueness, he insists that it is one source of incoherence in language. The two characteristics that make a vague expression inconsistent are the following. First, if an object x satisfies the expression '*F*', then so does another object y that is only minutely different from x in the relevant respects. This condition characterizes what it is for an expression to be *vague*. Objects which are close to a paradigm *F* are also *F*, and there is no definite limit as to how far from the paradigm an object must be in order to fail to be *F*. Second, vague expressions discriminate things: for any entity x that is *F*, there is -actually or possibly- another, y that is not-*F*. Together, both conditions create the circumstances that generate contradictions (c: 178, 180-183, 219). There is no vagueness without contradictions.

> The only way for... vague terms to be vague is for them to be inconsistent (c: 199).

This is clearly revealed in the sorites (b: 136).

In order to show that the common noun 'stone' is incoherent, we can construct a sorites, whose first premise lays down the existence of a stone. This is the premise that will be reduced to the absurdum. The major premise will embody the first feature of vagueness, asserting that, if something is a stone, then the removal of one of its peripheral atoms, or only a few of them, in the manner that is most innocuous and most favourable to the existence of a stone before us, will not make a difference as to whether there is a stone there or not. This major premise expresses a causal relation between events in the underlying dimension *G*, and processes in the supervening property *F*. Unger repeatedly affirms that this characteristic of vagueness presupposes the gradual nature of the world, in the sense that there is no breaking point at which the gentle removal of an atom produces the disappearance of the stone (a: 239, 248; b: 136). There are gradations in reality (b: 125; c: 199, 204). If this is so, then, eventually we will arrive at a stage where we have left only a few atoms, even one or zero, yet by force of the argument, that "agglomeration" shall constitute a stone. But this is absurd. Therefore, by *reductio ad absurdum*, the first premise must be forgone: there are no stones. This is an indirect proof of the inexistence of vague objects. Unger calls it the *sorites of decomposition*. To avoid the absurdity, Unger surrenders the existence of real vagueness.

On the other hand, there is also a direct proof of the same nihilist conclusion, the *sorites of accumulation*. If we begin with a situation where we have zero atoms before us, or only a few, we confidently can assert that there is no stone before us. Now, the major premise comes in: if something is not a stone, the addition of a single atom is not going to construct a stone. The force of the argument lies here: to augment a single atom to something that is not a stone will not create a stone; the simple addition of an atom to what is not a stone will not make the difference between what is not a stone and what is one. To oppose this premise is to expect a *metaphysical miracle*: to believe that an atom could make a stone. The long term effect of this premise is that no matter how skilfully or carefully we aggregate atoms, one at a time, we will never manage to assemble a stone. That is, in stepwise fashion, the reiteration of this premise will make us conclude that, independently of how many billions or trillions of atoms we have added, and regardless of the appearance of what we have before us, the outcome of the successive increments ought not to be a stone. Therefore, nothing is a stone.

So, Unger thinks that the nihilist claim is justified: there are no ordinary things, no people, neither vague objects nor vague properties. It would be a misunderstanding of his arguments to believe that they concern only words, for they are about things (b: 147). There are no *borderline cases*, cases that neither definitely satisfy the vague term nor definitely fail to satisfy it. Borderline cases would be those between positive and negative cases; but there are no positive ones; therefore, there are no borderline cases (c: 189). Now, let me add a proviso. Unger makes a distinction between ordinary objects (natural or man made: a table, pieces of furniture, a lake, a mountain, a rock, a person, a planet, etc.) and physical objects, of different shapes and sizes. And he claims that the nihilist assault is not applicable to the latter. This means that no sorites will be available to conclude that there are no electrons, or atoms or molecules. Consequently, the scope of Unger's nihilism is restricted: the physical reality does not come into question. But objects of common sense must be dismissed.

Thus, it is clear that Unger upholds the soundness of the sorites. For him, the premises are authentically true, and the form of the argument is valid.

In his endeavour to buttress the major premise of the sorites, Unger holds that its refusal implies what he calls the *miracle of conceptual comprehension*, i.e, the belief that vague words are extremely precise. For example, in the sorites of decomposition, if the removal of one atom brought about the disappearance of the stone, the noun 'stone' would have such an exactitude that it would be sensible to the atomic level. This is incredible, Unger complains. Vague words are not that precise. Exactly this is the objection Unger levels against many-valued logics. One could not expect that a minute change in the parameter *G* will affect the status of the supervening property *F*. The niceties of degrees of truth are irrelevant here. Indeed, for him, truth is not vague, but absolute (b: 153, n. 9).

It is important to note that concerning the question of the transition, namely, at what moment in the process of removal of atoms the stone stops being a stone, Unger answers that the transition takes place at no stage, since we do not have a stone to begin with. The process of destruction cannot start. There cannot be any transition from one opposite to the other, simply because there are no opposites in the first place. Thus, there are no transitions (a: 245; b: 136).

We have seen that the sorites, in both forms, direct and indirect, purportedly demonstrates the incoherence of vague terms: these apply to what they do not apply. And, since this is impossible, according to Unger, it follows that there is nothing to which a vague expression can be attributed. Hence, vague expressions do not refer at all, or are empty; they do not apply to anything. In a nutshell, if there are vague terms, they are incoherent; vagueness and incoherence go together. Thus, the denial of incoherence entails the eradication of vagueness. If vague words were consistent, then they would be precise. But precision does not fit vagueness. Unger time and again maintains that the only rational, or adequate way to respond to the incoherence is nihilism.

To end this overview, let us see some sombre consequences Unger draws from his nihilism. First and foremost, it follows that no sentence in which a vague term occurs can be true (b: 148). It is in the spirit of his philosophy to assert that none of the three next sentences make much sense: 'there are no stones', 'there is a stone', 'all things are stones'. Moreover, this first result prohibits sentences describing "events" involving vague objects. There could not be, for example, the eruption of a volcano, for a volcano, being inconsistent, does not exist. Secondly, his position underpins skepticism, since it certainly follows from nihilism that no beliefs about common sense objects can be true. Therefore, our thought and language would have much fewer contacts with reality than it is usually conceded (b: 149).

2a.i.- Criticism

It is time now to take stock of Unger's conception.

First, we must resolutely agree with Unger that fuzziness is indeed contradictory, for it hosts two characteristics that are in conflict: its *diffuseness*, i.e., its spreading itself along the sorites series, being applicable to objects differing only minutely from the paradigms, and its discriminatoriness. Fuzzy terms are diffusive and discriminatory, and hence give rise to real contradictions. This is as it should be. Unger deserves to be praised for his urging that fuzziness requires the recognition of contradictions. This will be his lasting contribution. We enthusiastically approve his pointing to two constitutive elements of fuzziness which do not coexist without contradiction. We join him in asserting that fuzziness is contradictory.

However, despite this initial sympathy for the contradictorial characterization of fuzziness, it is imperative to remind ourselves of the different attitudes towards contradictions. The problem is how to react to them. Unger is doomed to see contradictions as something very bad, like a plague. He says that the only rational response to contradiction is nihilism (b: 140). He cannot but reason by *reductio ad absurdum*: the supposition of the existence of a vague object generates contradictions; but contradictions are incoherent and impossible; therefore, there is no fuzziness, neither in words or concepts, nor in things. Yet he is wrong on this point. Embracing contradictions is a rational option, as long as there are paraconsistent logical systems that are not trivial[18].

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18 A system is *trivial* when every sentence can be derived from its axioms; when every well formed formula is a theorem of it.

What we need to have is a criterion that allows us to sift benign contradictions from absurdities. In fact, we set simple contradictions, "p and not p", apart from supercontradictions, "p and not p at all", according to the kind of negation involved. Fuzziness, inasmuch as it is diffusive and discriminatory, does induce harmless contradictions; but it is not responsible for the absurd conclusions derived in the sorites. We have tall short men (b: 130). To accept this does not compel us to admit also that an "agglomeration" of zero atoms is a stone. This is nonsensical. Logic should be modified to dismantle the paradox.

Unger is not explicit about the formal details of the sorites argument, downplaying the role of logic. For example, one wonders how exactly the general conclusion of the sorites of accumulation is arrived at. A solution in mathematical terms or using technical devices is out of place in the problem of vagueness. When he mentions mathematical induction, it is only to dismiss it as playing no part there (b: 128, 133). He moves at the intuitive or informal level, being concerned rather with the physical process of slicing a table, dividing it, and reducing it to nothing. But whenever he alludes to the major premise, he gives it a conditional form. If a_i is F, then a_{i-1} is also F; and if a_i is not F, then a_{i+1} is not F either. So, I think that he will interpret the major premise as the Preservation Principle, (Pre.P): $Fa_i \supset Fa_{i-1}$, and correspondingly, the rule of inference is *Modus Ponens*.

If the argument proceeds by MP, we agree with Unger in considering it valid. Yet, one instance of the conditional major premise must be entirely false. In the sorites of decomposition, there must be a number of atoms which do not constitute a stone. What is this minimal number? A number n greater than zero will not stand scrutiny, since the sorites is such that we can prove that n atoms suitably arranged constitute a stone. It is an indisputable fact, and beyond doubt that zero atoms are not a stone. But, then, we have the last conditional: *if* one atom is a stone, *then* zero atoms are also a stone. This is completely false. We may concede that the protasis is true, for the sorites allows that much, but the apodosis is downright false. Thus, what is wrong with the conditional sorites is one instance of its major premise. The existential premise can stay; there is no need to drop it. Ergo, the argument is not sound.

Unfortunately for Unger, he cannot distinguish the (Pre.P) from the Parity Principle, (Par.P): $\sim Fa_i \vee Fa_{i+1}$, or from the Continuation Principle, (CP): $\sim(Fa_i \wedge \sim Fa_{i+1})$. But we have seen in the Introduction that they are not the same. Unlike Unger, we can give up the (Pre.P) and at the same time, keep the (Par.P) and the (CP). This is why we can renounce the conditional major premise without sacrificing fuzziness, since we still have other means of conveying the diffuseness.

So, we have concluded that there must be an atom whose removal produces the complete destruction of the stone. Is this a breaking point? No, because we also acknowledge that, before arriving at the stage where not-F is exemplified absolutely, there are preceding objects which are also not-F. Thus, there is continuity. The gradualness of reality is not lost.

I want to add two final remarks to end this assessment. One serious inconvenience of Unger's nihilism is that he seems to be condemned to *ineffabilism*. That is, if his theory is true, he cannot express it. In effect, we can assert the thesis of nihilism by saying that:

> «typical» vague expressions, like 'stone', 'table', 'person', and «many other» terms, are inconsistent in virtue of their being vague (c: 178, 208, 216); consequently, there are no entities to which they apply, and those terms are empty; hence, «virtually all» of our common sense beliefs are untrue (a: 249).

But according to his criterion, no sentence containing vague terms can be true. So, he is debarred from conveying the supposed truth of his own theory.

Lastly, Unger charged many-valued logics with a commitment to the miracle of conceptual comprehension. If an atom is removed from a stone, that must affect the attribution of the word 'stone' to whatever we have before us. This is indeed so. Small degrees matter a little; we cannot ignore them. Of course, the effect of a minute change in the dimension *G* on the property *F* must be commensurate. That is what the *G-F* Correspondence Principle teaches us. And we saw that the apparent problems this principle faces are dispelled in a paraconsistent logic. Thus, our acquiescence to it is justified. We regret that Unger, who has perceived the graduality of the world, has not assumed the (*G-F* Cor.).

2b.- Horgan

Terence Horgan's position is complex. If we would like to classify him in some school, there are three plausible candidates: incoherentism, indeterminism, and nihilism. He himself has called his stand *transvaluationism*, with an eye to dissociating it from standard nihilism. The label signifies several things. Primarily, it means that we should go beyond the idea that an assignment of different truth values to immediately neighbouring sentences of a sorites sequence respects vagueness. That is, he thinks that not to assign the same alethic status to them -which is the recommended policy of many-valued logicians, agnosticists and supervaluationists- necessarily violates vagueness, since it introduces a sharp precisification. Moreover, we should move beyond any overall valuation. In another sense, 'transvaluationism' means something more specific, concretely the thesis that vagueness is a benign incoherence. Let me elaborate on this fundamental claim.

Horgan favours a conception of vagueness which sees it as being logically incoherent, and nonetheless possible and meaningful. His ground for affirming the inconsistent nature of vagueness is comparable to that of Unger, i.e., vagueness is composed of two mutually unsatisfiable conditions, namely, *boundarylessness* or *robustness*, and the difference condition. We have met the former in the introductory chapter; it refers to the absence of a limit between any two contiguous members of the soritical sequence, so that the status of the first element of the series is extended to all others. He takes it to be a fact that there is no minimal height for a person to qualify as tall. The norms governing the use of words do not justify any precise boundary. Vagueness is diametrically opposed to a sharp precisification. This first feature is related to the conditional major premise of the sorites: the robustness of vagueness entail the attribution of the same semantic status to adjacent members (1994a: 174). And the second feature, the *difference condition*, is that vague terms discriminate things: some are *F*, others are not. In the sorites sequence, at the beginning, the elements are *F*, but they are not-*F* at the end. Hence, when we conjoin both properties of vagueness, its being boundaryless and discriminatory, then we get contradictions. Frank is bald and not bald; a heap is not a heap. For Horgan, incoherence is a legitimate and indispensable part of vagueness (*Ibid*.: 179-180; 1994b: 106-107, 115; 1998b: 313, 315, 325). The task of a theory of vagueness is not to eliminate inconsistencies but to control, or to isolate them. For him, this logical incoherence constitutes an invitation to go beyond classical logic. There is no coherent logic of vagueness (1994a: 176, 181).

We need to add two qualifications. For Horgan, the acceptance of contradictions is limited to the level of language and thought, but does not reach the ontological level. Fuzzy words and concepts are contradictory for they lack boundaries and are discriminatory; yet, there are no vague objects or properties in the mind independent, discourse independent world. This is impossible, Horgan says. Furthermore, the two irreconcilable qualities of vagueness are not on a par, but are in a hierarchy, the discriminatory condition dominating the boundarylessness. This means that, at the moment of dismantling the sorites paradox, one has to put a question mark on the major premise. Because of this dominance relation among the two basic traits of vagueness, Horgan claims the incoherence is disciplined, obeying the orthodox standards of classical logic. There is no rational ground to be in panic.

Concerning the sorites, Horgan has two attitudes. On the one hand, he regards it as a sound argument[19], demonstrating that there are no vague entities in reality. If we take the heap paradox as making reference to ontological vagueness, then the contradiction arrived at shows that one of the premises must be wrong. The premise to be reduced to the absurdum is the minor one, which establishes that the paradigmatic instance of *F* is *F*. It is «not true» that the person who has zero hairs on her scalp is bald! (1994a: 165). There are no red objects, no tall persons, no heaps, etc., nor baldness, tallness, and so on. In virtue of this denial of the ontological commitments of the vague talk, Horgan accepts that his position is a sort of nihilism. But he prefers to call it '*Parmenidean materialism*'. The reason why he avoids a pure nihilist stand is that he concedes the truth of some vague predications. But, how can a sentence predicating a vague property of an object be true if the only existing properties are precisely demarcated? What explains this discrepancy is his peculiar semantics, in which nouns and adjectives are meaningful independently of their reference relation to something objective, and truth itself is not a direct language-world relation.

On the other hand, Horgan's unique aim is to eschew the conclusion of the paradox doing justice to the robustness condition of vagueness, i.e., without postulating any arbitrary precise boundary to the vague concept. He thinks that, to comply with robustness, it is sufficient to avoid actually falsifying the major premise, which is achieved -according to him- provided that the major premise is never declared false. But, he contends that not to be false does not mean to be true. The Principle of Bivalence has to be revised, specifically the presumed exhaustive aspect of the truth values: the absence of the one does not entail the presence of the other. So, the major premise is not false but it is not true either; therefore, it is indeterminate. Horgan promotes a suspension of judgement concerning the assignment of truth values to the sentences of the soritical series. He refuses to make a specific overall assignment. Remind that the name 'transvaluationism' intends to reflect this global non assignment. He alleges that there is no fact of the matter concerning how the semantic transition from one contradictory to the other is made, and this is its robustness, the essence of genuine vagueness (1994a: 163). There is an ontological indetermination.

2b.i.- Criticism

There are several points in Horgan's theory calling for a critical remark.

First, let me comment on Horgan's incoherentist conception of fuzziness. In this regard, as it was the case with Unger, I do sympathize with the general claim made that fuzziness is inherently contradictory. There is a real tension between the major premise of the sorites, and the fact that the first members of the soritical series are *F* whereas the last ones exemplify the opposite. But, how could there be a transition if every pair of adjacent elements must receive the same semantic status? For, to the extent that the major premise is true, one cannot pass from *F* to its opposite; and vice versa, to the extent that the first and last members exemplify contrary qualities, the major premise is false. Horgan enunciates the dilemma thus: either all members are *F*, or there is a sharp partition. So, fuzziness has two characteristics, one being the negation of the other.

However, if we have a paraconsistent system, there is no need to choose. We can have both, robustness and the discriminatory condition. If we had to sacrifice one, I think the option more in accord with fuzziness would be to eliminate the second, for, as the slippery slope shows, despite the omnipresence of *F*, things are differentiated by the degree of their possession of *F*. Some balls are very close to A, others are less close, and others further over there are still much less close. Thus, even if all entities possess a property, not all of them possess it to the same degree, and therefore they are distinct from each other.

If we have to drop one of the components of the contradiction to restore logical discipline, the ensuing position is not paraconsistent any more, nor incoherentist either. A

19 A *sound* argument is one with true premises and a valid rule of inference.

contradiction comes into existence only when both the thesis and its negation are jointly asserted; and it is destroyed as soon as one contradictory has to go. As a matter of fact, I do not see what Horgan gains by admitting contradictions only in language and thought but not in the world. On the contrary, that attitude of allowing contradictions in the subjective sphere while keeping reality free from them can only make him distance from a realist semantics, for he loses a relation of direct reference and truth. Supposing that reality contains only objects and properties sharply divided, without contradictions, why do we want to have true fuzzy predications? And, under the same supposition, if our talk and thought are fuzzy but not the world, then we would have a completely distorted representation of reality, as deceiving as that of the contrary hypothesis, where the entities are fuzzy, but words have neatly carved meanings. Thus, Horgan's halfhearted admission of contradictions is more harmful than advantageous.

Were Horgan wholeheartedly committed to accepting contradictions, he could do away with his nihilism. If he genuinely embraced logical incoherentism as something beneficial, then actually he would be removing all obstacles to contradictions in reality. He would be entitled to posit bald persons, and tallness, etc. as legitimate dwellers of the world.

On the other hand, fuzzy entities are not boundaryless, as was shown in the Introduction. They do have limits, and indeed many.

Summarizing this primary point, I take Horgan's contradictorial conception of vagueness as an advance in the debate, as a step forward in the right direction. However, he has not gone far enough. Incongruously, in the final analysis, his position is not different from agnosticists or supervaluationists[20] inasmuch as they all end up denying ontological fuzziness from their rejection of real contradictions.

Second, let us examine if the reiterated goal of Horgan's project is served by his indeterminism. Recall that he wants to defuse the paradox respecting vagueness' robustness. As a matter of fact, he proposes this as a methodological constrain any theory of vagueness should meet. *Robustness* is explained in negative terms: there is no precise, abrupt transition from one property to its opposite (1994b: 106; 1998b: 314). So, Horgan is implicitly demanding to block the conclusion of the paradox without relinquishing any major premise, and personally I adhere to this sound requirement. However, Horgan himself does not sustain the truth of all major premises. He certainly abstains from saying that they are false. But since they are indeterminate, they are kept with a question mark, neither asserted nor denied. I think this indeterminist attitude does not lend proper support to the major premise. A positive underpinning is in place only when one attributes a designated truth value to the premises. If we do not take a stand neither in favour of, nor against both, the major premise and a sharp partition (1994b: 113), then we are putting both of them on the same footing. The major premise could be true as it could be untrue. If that is all the patronage one can muster for the major premise, that seems very little, amounting to almost no backing. So, I conclude that Horgan's indeterminism does not live up to his own yardstick.

Third, and finally from the point of view of a many-valued logic, something that Horgan has said is ambiguous. One of the criticisms he makes to many-valued logics is that not to assign the same alethic status to contiguous members in a sorites sequence is to introduce a sharp boundary. For example, if the truth value of "Fa_i" differs from that of "Fa_{i+1}", then there is a cutoff point between them. But, in some many-valued, or fuzzy logics both sentences can have different truth values and, nonetheless, both can be true or both can be false. That is, the expression 'alethic status' of a sentence "p" can be understood in two ways.

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20 I include supervaluationists among those that deny ontological vagueness because supervaluationism takes vagueness as a semantical phenomenon: entities are sharply demarcated in reality; a predicate '*F*' is vague in that it is indeterminate which of the many sharp properties is the referent of '*F*'. Vide: Varzi 2003c; Bittner and Smith. Williamson 2003: 702-4; Edgington 2001: 377; Heller 1996: 181-2, including note number 7.

It can mean the specific, exact truth value assigned to "p" (commonly a number taken from the unit interval), or it can refer to whether the truth value is either designated or antidesignated or neither designated nor antidesignated. Having diverse truth values is no impediment to having the same alethic designation. Thus, the similarity principle is not violated. Both "Fa_i" and "Fa_{i+1}" can be true or both can be false, and nonetheless, each can have a different truth value. That there is a soft boundary between any two adjacent members should not be any source of concern either, as we have seen in the Introduction.

3.- Paraconsistent Approaches

The present section will concentrate on proposals which unfortunately have for the most part been neglected in current discussions: those which appeal to a paraconsistent logic in order to defuse the paradox and to elucidate the underlying phenomenon of fuzziness. Furthermore, solutions allowing for contradictions in this particular domain do not abound; and those which do exist have not received the attention they deserve. As a token of a beforehand refusal attitude, let me quote a passage from Rosanna Keefe (2000: 197):

> ...many philosophers would soon discount the paraconsistent option (almost) regardless of how successfully it treats vagueness, on the grounds of the... absurdity of p and ¬p both being true...

(I have left the negation sign as it appears. But it is clear that '∼' should be there instead). But, even if, at the end, one is going to reject all paraconsistent perspectives, that dismissal should be the result of a detailed assessment of pro and con reasons; in the absence of such a canvassing, and in the light of arguments given in favour of one of those systems, that dogmatic attitude does nothing to make the current discussion move on. Anyway, in this section, I will give an overview of three approaches tolerating inconsistencies: subvaluationism, Australian relevantism, and adaptive logics. I will critically examine them in turn.

3a.- Subvaluationism

The application of subvaluationism to the problem of fuzziness and the sorites has been studied by Dominic Hyde [1997], who does not endorse it. The theory shares with supervaluationism the use of precisifications, inheriting thereby its problems. According to subvaluationism, fuzziness can be clarified by assimilating it to a kind of ambiguity, that is, a fuzzy sentence can be interpreted as expressing several different exact propositions, its truth value being dependent on the admissible ways of disambiguating it, or of making it more precise. A sentence attributing a fuzzy predicate to a borderline case can have a true as well as a false disambiguation; hence, a fuzzy statement can be both true and false, because true in a precisification and false in another. At the root of this conception, there lies a notion of truth weaker than that used by supervaluationism, in that it is not required that a sentence be true in all admissible precisifications in order for it to be true *simpliciter*, just one precisification suffices. So, what were indeterminate sentences in supervaluationism become (weak) inconsistencies in subvaluationism. Vagueness gives rise to truth value gluts (647, 649). However, although on the one hand, a sentence, "p", and, on the other hand, its negation, "¬p", can both be true, their conjunction, "p∧¬p", is not true. Thus, fuzziness is rightly seen as contradictory: a predicate applies, and does not apply to its borderline case. But, the adjunction rule fails: from p, q, you cannot infer p∧q. This is a peculiarity subvaluationism shares with Jaskowski's discussive logic. So, properly speaking, there are no true contradictions in the system, but just two contradictory statements, both true, but separately. The theory is weakly inconsistent: it countenances inconsistencies without contradiction.

As for the solution to the sorites, Hyde discusses a form of the paradox in which its major premiss is construed as a conditional, and the inference proceeds by means of *modus* *ponens*. It is precisely this rule that is not *valid* according to subvaluationism, i.e., the argument does not preserve truth simpliciter.[21] To see this, suppose: (a) that the first member of the soritical series is a heap; (b) that the removal of a grain does not make the difference between what is a heap and what is not, i.e., that any two adjacent grain piles satisfy the condition that, if the first is a heap, so is the other consisting of one less grain; and (c) that the last member of the series, eventually, say, 0 grains, is determinately not a heap. Granting all these three assumptions, there is no way to block the conclusion (that a "collection" of 0 grains is also a heap) other than by repudiating the inference rule: *modus ponens* is invalid, for although the premisses are true on some disambiguation, the conclusion is false on all disambiguations.

So far the strategy is clear: the soritical conclusion does not follow because the argument form is invalid. Now, Hyde tries to explain the failure of *modus ponens* by appealing to the conception of vagueness as a kind of ambiguity, as previously seen. What is wrong with the sorites is that it commits the fallacy of equivocation: the premisses are not true on the same disambiguation. The same sophism is incurred as in the following argument: $\Diamond p$, $\Diamond(p \supset q) \vdash \Diamond q$; what is required here for the truth of the conclusion is that the same possible world encompassing both p and $p \supset q$ comprise q also. Analogously, *modus ponens* demands that the same disambiguation make all the premisses and the conclusion true. However, subvaluationism only requests that the premises and conclusion be all true on some disambiguation, but not necessarily the same. But if this is so -Hyde concludes-, the notion of validity in subvaluationism is too loose, and should be appropriately strengthened in order to fit the demanded standard.

What do we think of subvaluationism? Well, it seems that to abandon *modus ponens* is too drastic. I do not claim that this rule is unproblematic nor that it is self-evident; nothing in principle escapes the possibility of being revised, as holist Quine has argued for; but dispensing with such a fundamental rule should be left as an extremely last resource, when all plausible alternative ways out are not available. Notwithstanding, the subvaluationist strategy to the sorites is on the right track: the major premises are true, but the reasoning is not truth preserving.

Moreover, it may be objected that the rule involved here is not *modus ponens* but disjunctive syllogism. Indeed, as we saw in the Introduction, the similarity of adjacent members in the soritical series does not support the conditional reading of the major premise, but the disjunctive reading (with weak negation). Hence it is debatable whether the argument is an instance of reiterated *modus ponens*.

Besides, as a consequence of the failure of adjunction, truth-functionality is lost for conjunction: the truth of both "p" and "q" does not determine the truth of "p and q". This failure -Hyde correctly says- might give ground to reject the subvaluational conjunction.

The main drift in Hyde's argumentation is to demonstrate that subvaluationism is on an equal foot with respect to supervaluationism; that none is superior, since the evidence supports gaps and gluts equally, and therefore, that there is no justification for the preferred status of supervaluationism. But looking at the choice from a broader range of alternatives, both options together may seem to be unsatisfactory, in view of the fact that whatever deviance is found at the level of non truth-functional conjunction and disjunction is going to be also transmitted to the meaning of the quantifiers. Therefore, if supervaluationism can be charged with not capturing the sense of the English expression 'there is', a similar charge may be levelled against the subvaluationist treatment of the universal quantification: "for all x, *F*x" may fail to be true although all its instances are true. This problem seems to disqualify both candidates, supervaluationism as well as subvaluationism, as logics and semantics of natural

21 More precisely, an argument is *valid* «just in case whenever the premises are true in some admissible precisification the conclusion is true in some admissible precisification» (647).

language. Hyde grants all this, distancing himself from subvaluationism. And I cannot but agree with him on this point.

3b.- Relevantism

The second paraconsistent approach to fuzziness and the sorites paradox that I want to review is that of relevantism as developed by Richard Sylvan and Dominic Hyde [1993]. They criticize most of the definitions of fuzziness for their failing to account for its over-determination, admitting at most its under-determination. An adequate characterization should acknowledge both. Indeed, due to one of the De Morgan principles, the negation of the principle of excluded middle, $\neg(p \vee \neg p)$, is equivalent to an indetermination, $\neg p \wedge \neg\neg p$, i.e., neither p nor its negation are true; and by double negation, this amounts to a contradiction, $\neg p \wedge p$. So, Sylvan and Hyde would accept that those indeterminists who, when queried about a fuzzy predication, affirm that the corresponding object neither is nor ceases to be *F,* but are unwilling to allow for contradictions must deny either De Morgan or double negation, the latter being the most likely candidate. Indeterminism comes at a price. By contrast, relevantism is prepared to embrace under-determination as much as over-determination.

Furthermore, the other aspect of fuzziness concerns the borders of the extension of fuzzy words. In this regard, Sylvan and Hyde describe the boundary as a region in which the exterior and the interior parts are neither exclusive -allowing for an overlap of *both*- nor exhaustive, including members which are *neither* in nor out.

Clearly, this enlargement of the bivalent possibilities moves in the right direction, and should be welcome by whoever is urging that the very narrow frontiers set once and for all by CL be expanded. However, having in mind that the Neither and the Both areas are logically equivalent, and thus amount to only one, it might seem that to concede just one more case beside the traditional two is not sufficient. Indeed, not all elements in this non standard region enjoy the same status. To make room for gray is good, but not everything in there is 50% black and 50% white. Mixtures of the extremes in different proportions should be permitted; in order to have the full range of cases, an infinity of intermediate situations must be granted. For more on this criticism, see Peña [1996].

Sylvan and Hyde say that fuzzy logic is not a rival of relevantism, since the latter has multi-valued matrices, and they even go as far as claiming that 'the more or less' is an essential feature of fuzziness, and consequently, this functor will be needed as a test of adequacy for any logic of fuzziness (p. 14). But they claim their system can accommodate the functor in the syntax without being committed to a many-valued semantics. There is no need for an appeal to degrees of truth. The authors give an instance of the kind of analysis they have in mind.

'Approximately p' is true iff that p is approximately true.

I assume that they would grant a more general principle, like

'...p' is true iff p is ... true

in which the blank space should be filled by an adverb of quantity, or intensity. For example,

'More or less France is large' is true iff that France is large is more or less true.

(The predicate '...is true' of the left member will be treated as redundant, so that the quotation mark device can be dropped, assuming a deflationist theory of truth). But if this is so, we are already embarked on the road to a many-valued logic. In the simple case of an attributive sentence, it is not clear how to interpret the left member of the biconditional other than as expressing the extent to which an object possesses a given property. The more natural way of understanding the functor 'more or less' is to take it as affecting the degree to which the object belongs to the extension of the predicate. Thus,

France is more or less large.

But to accept this seems to commit us to accepting fuzzy sets. Put otherwise, we have found that degrees of truth go hand in hand with degrees of possessing a property. More exactly, "both" kinds of degrees are strictly identical. In general, a sentence predicating *F* of *a* is true to exactly the same extent as *a* is *F*. This is how the functor of weak assertion 'more or less' can be handled in a many-valued fuzzy logic. But unfortunately, Sylvan and Hyde do not elaborate on this issue in the paper.

As for the sorites, the authors also consider its conditional form; the type of argument using the 'indistinguishable' predicate is reducible to that conditional scheme. The difference in relevantism between two kinds of conditionals: the if...then, here represented as $\supset$, and entailment, $\rightarrow$, does not affect the uniformity of their approach, since parallel considerations apply to both. What they want to say in this connection is that *modus ponens* for the conditional $\supset$ is not truth preserving when it is reiterated a large number of times -as it occurs with the sorites-, though it is *locally* valid, i.e., for a small number of reapplications. Just see what happens with the segment of the spectrum of colours flanked by yellow and red, which is so constructed that the members of any pair of contiguous bands are indiscernible from each other:

$$a_i \sim_d a_{i+1}.$$

To be faithful to this fact entails to retain the truth of the major premise. Yet, the first band, a_0, is yellow, but the last one, a_z, is not; a_0 and a_z are discernible:

$$\sim(a_0 \sim_d a_n).$$

Therefore, the relation of discernibility fails to be transitive. And similar considerations apply to the conditional. In a soritical series we have:

$$Fa_0,\ Fa_0 \supset Fa_1,\ ...,\ Fa_{z-1} \supset Fa_z,\ Fa_z.$$

But, since all the members of the chain are true, except the last, iterated *modus ponens* does not hold. As we mentioned, the authors distinguish between *validity* and truth preservation; the former is defined syntactically: whenever "p" together with "p $\supset$ q" are proven, so is "q". The validity of *modus ponens* is not questioned, but only its truth preservation in the long run.

This is in a nutshell the position of the Australian philosophers. They both deserve to be praised for their defence of a paraconsistent conception of fuzziness and their demand for a reform of classical logic. This notwithstanding, the failure of *modus ponens* to preserve truth is too high a price. As previously said, insofar as one theory drops a classical rule of inference, it places itself in a disadvantage position in comparison with another theory keeping -under certain interpretation- all rules of CL. The system advocated in the present monograph, has the advantage of being strongly conservative, in the sense that it retains all classical principles and rules of inference, and in that respect, it is superior to relevantist logic. Additionally, it contains a non classical implication whose properties closely resemble relevantist entailment.

3c.- Vagueness-Adaptive Logic

The third and last paraconsistent contribution to the debate I want to examine is the application of adaptive logics to fuzziness made recently by Guido Vanackere and Bart Van Kerkhove. The latter reinforces the pragmatical approach of the former, who belonged to the team of researchers developing aspects of adaptive logics found by Diderik Batens of Ghent. So, I first give a rough outline of these logics in general, as a background, based on Batens' articles listed in the Bibliography.

The main idea is that many theories, scientific or otherwise, sometimes at one stage of their development come to collide with a presupposition of their underlying logic, usually CL. The thesis that goes against some of the principles or rules of the logic is called an abnormality, and it always takes the form of an inconsistency, which therefore threatens to render the whole theory trivial. In these circumstances, an *inconsistency-adaptive logic*, IAL, comes to the rescue. IAL is a provisional tool allowing to reason from the incoherent theory in order to locate the inconsistencies, with the ultimate purpose of eliminating them and of arriving at a new theory. But, instead of definitely invalidating the rule of inference employed in the derivation of the abnormality, the approach used is to consider the instantiations of such a rule as incorrect, so that the rule is suspended relative to the set of propositions giving rise to the inconsistency, but this does not prevent the rule from being used in other contexts where no abnormalities arise. Rules are valid as long as and provided that no contradictions are derived with their help; whenever a sentence of the form "$p \wedge \neg p$" might be deduced, the rule used is no longer valid in that context; therefore, rules are conditionally valid. Adaptive logicians believe that it is preferable to "locally" invalidate the rule rather than to allow for inconsistencies; this is their way of avoiding trivialization. An adaptive logic thus plays its proper role during the transition period. It is weaker than the *upper limit logic* (ULL), i.e., the intended logic underlying the theory, but it is stronger than the *lower limit logic* (LLL), which results from entirely dropping the principle conflicting with the abnormality. In the case of the IAL, the LLL invalidates the principle *ex contradictione quodlibet*, and in so doing, LLL becomes a paraconsistent logic. In other words, the AL does not have all the consequences of the original ULL, but it is more powerful than the LLL. Inconsistencies are somehow tolerated, at the price of allowing for exceptions to the rules, but consistency is considered a sound methodological requirement.

In order to apply this framework to the treatment of fuzziness and the sorites paradox, Vanackere and Van Kerkhove need to consider fuzziness as a kind of ambiguity. The supposition that a fuzzy expression has a unique meaning is -according to them- what causes the paradox. To see this, consider the following version of the sorites which nicely brings to the fore the close relation between fuzziness and degrees:

John is rich;
John is a little bit richer than Mary
∴ Mary is rich too.

Leaving aside the comparative, we have two occurrences of the monadic adjective, namely, 'rich' and 'rich too', both of which, normally, would be considered as having the same meaning. But the authors think that precisely this would be a mistake; if we want to avoid identifying the rich and the poor, we better distinguish the several occurrences of the monadic predicate by giving each a different index. So, let us symbolize the argument as follows:

$$R^1j,\ j >^{\mathrm{R}} m \therefore R^2m.$$

(I have substituted the original '<' for '>'). The next argument in the chain is: Mary is rich too; Mary is a little bit richer than Paul; then Paul is rich too. That is:

$$R^2m,\ m >^{\mathrm{R}} p \therefore R^3p.$$

And so on. Were we to take R^n and R^{n+1} as identical, we will end up being caught by the paradox. So, each occurrence of the predicate has a different sense. And this is part of the solution to the paradox. We would be forced to renounce the assumption: "one term, one meaning"; in the presence of vagueness, this must be given up.

Finally, let us see how the *vagueness-adaptive logic* (VAL) works. We need to identify which abnormality is in play here. It is any sentence of the form: $\neg(R^n a \equiv R^{n+1} a)$. This is abnormal in that it violates the axiom of the ULL of meaning univocity. And precisely this plurivocity is an instance of vagueness. Hence, fuzziness is treated as an abnormality.

Now, the main problem I have with this conception is its considering vagueness as a sort of ambiguity. Ordinary speakers will agree that the sense of the vague predicate 'rich' does not change because the financial situation of the person varies in one cent more or less. We would be more inclined to say that Paul and Mary are rich in the only sense of the word, differing by a certain amount of money, rather than saying that both are rich but each in a different sense of the adjective. If John is rich_1, Mary is rich_2, Paul is rich_3, then the word 'rich' would denote an exact amount, which -I take it- goes against the unique denotation of the adjective. It seems to be a presupposition of the paradox that the meaning of the predicate remains constant from beginning to end, so that when it is affirmed that a_0 is *F* and that a_z is not *F*, the meaning of *F* has not been altered in both assertions, nor in any of the intermediary cases. A central ingredient of the VAL conception is that a pair of contradictory sentences, p, and not-p, result from ambiguity. This appears questionable from a point of view which sees fuzziness precisely as contradictory.

The merit of any IAL view is that it permits to reason within contradictorial theories averting triviality. However, there are other systems that can fulfil the same function without suspending any classically valid rule of inference, as it is the case with *Aj* and *Aq*.

In conclusion, the three paraconsistent theories we have studied are correct in allowing contradictions produced by fuzziness, but divergences surface as a result of differing ways of handling contradictions within the frameworks of rival systems. There will come one day when some theory of this type will be assessed as the closest to satisfy the standards of adequacy.

4.- Raffman's Contextualism

In [1994b] and [1996], Diana Raffman elaborates a contextualist approach to vagueness and the heap paradox. We give here the basic ideas of her theory.

Raffman's point of departure is an astonishing feature of the soritical series: that there is a categorial difference between its end points although its adjacent members are only *marginally different*, i.e., indistinguishable or such that their difference is barely noticeable. For example, consider a series of 50 coloured patches going from red to orange. How could a difference in kind arise in the series if there is no non-marginal difference between contiguous elements? This is a particular version of the *transition problem*. It seems that the indistinguishability of neighbouring members prevents the placement of a dividing line anywhere in the series. But, since the extremes of the series are different, there must be a borderline somewhere in the sequence. This is related to the «fact» that, if we ask a person to classify the items of the soritical series that begins with a member that is *F*, the person will stop applying the predicate '*F*' at some point. So, the problem reformulated according to the fact just mentioned becomes that of explaining why a subject may classify adjacent members in different categories despite their indiscriminability. Put the question in other words, Raffman asks herself why the major premise of the sorites can be false (1994b: 50). Yet, vague words, not having sharp boundaries, preclude the occurrence of a crisp division. If there is going to be a satisfying solution to the difficulty, it must respect the blurriness of the boundaries. Thus, this is part of another datum of the problem: «it would make no sense to postulate a *boundary* of any sort between...» marginally different members (1996: 176). By a '*boundary*', Raffman understands a fixed division between adjacent members into incompatible kinds relative to the same context (1994b: 57; 1996: 176).

Raffman's answer to the question contains several components that in her opinion contribute to save the «apparent», «visual», phenomenological continuity of the series (1994b: 49, n. 14; 51; 1996: 179). First, she draws a distinction between two ways in which one can judge the members of the series: *pairwise* or *singly*. The former sort of judgements, called '*discriminatory*', compares adjacent elements, and the latter sort, named '*categorial*', compares one item of the sequence with a «prototype» (1994b: 48). Pairwise judgements tend to overlook the change in the series as it progresses from one extreme to the other; whereas single judgements are made in abstraction of the contiguity of the members of the series. Categorial judgements could be made in any order, and without presupposing that the elements of the sequence are placed next to each other. The distinction between pairwise and single judgements renders its services in the following manner. It is never the case that the major premise is false when the consecutive members of the series are judged pairwise; but the possibility arises that, when they are judged singly, the premise is false.

Second, the categorial shift is not a border crossing, for «the categories... live in the mind» (*Ibid*.: 45). That is, the change from one category to the other is a purely psychological phenomenon (1996: 189). Raffman says:

> the only difference between circumstances in which... #43 is red and circumstances in which it is non-red lives in the psychology of the competent judging subject. *Ex hypothesi*, no variation occurs in the "extra-mental" world (1994b: 65).

To speak of an object as looking red amounts to speaking of it as being judged to look red. In general, Raffman adopts a *response dependent* analysis of any vague predicate: it applies to some objects when and only when they are being so judged (*Ibid*.: 67, n. 28). Again,

> once... -... all but the categorial- contexts have been fixed, an item lies in a given category if and only if the relevant competent subject(s) would judge it to lie in that category (*Ibid*.: 69-70).

The application of a vague predicate varies with the psychological state of the subject (1996: 181). So, part of this second strategy to solve the problem consists in relativizing the categorizations made by subjects to mental points of reference, or *anchors*.

Despite this psychologist turn taken by her approach, Raffman warns that she does not mean to reduce the property attributed to the object to something dependent on the subject. She is not providing a meaning analysis or the intension of vague expressions. Rather, what she is relativizing to psychological contexts is the extension of vague predicates (1994b: 66).

If a predicate applies to an object relative to a psychological context, which is a state of our mind-brains, then it results that a classification of all members of a soritical series such that the first are *F* while the last are not *F* must be made relative to two different internal contexts. In fact, if one begins to classify the items in the series from the red end, one is biased to look the subsequent members as also red, so that, if a change occurs eventually, that should be so because of a psychological change in the subject. In other words, at the moment of a category shift, a switch of mental anchor occurs. And of course, where the turning point happens varies from person to person, and for the same subject from one occasion to another. For instance, suppose that, on a particular episode, the transition takes place at patch #27. From the point of view of the initial internal context, IC1, patches #1-26 look red, and #28-50 look orange; patch #27 has no collor relative to IC1. However, at any time, for any patch, there is some total context (that includes an internal and an external context) with respect to which the patch is coloured. So, with respect to the initial total context, TC1, #27 is not red. When patch #27 is judged to look orange, that is so relative to another internal context, IC2, and therefore, with respect to another total context, TC2. Thus, an opposite judgement concerning adjacent members is made, but not relative to the same anchor; #26 is red and #27 is orange, but not with respect to the same point of reference (*Ibid.*: 67-9). There is a variable, non fixed boundary but in relation to different total contexts (1996: 180).

Once the shift has been effected at patch #27, the predicate 'orange' starts to spread backwards, and then #26 tends to look orange. Before the change, #26 looks red; after the change, it looks orange. Hence, the extension of the predicate varies with the context.

Again discontinuity is avoided. At the time #27 looks orange, one does not want to assert that #26 is red, on pain of inserting a sharp dividing line between the two patches. And effectively, #26 does not look red, because -Raffman claims-, at that moment, #26 is not being judged.

And in the third place, a crisp borderline is averted, for the psychological shift of context does not have a semantical force. It is not mandatory, since «there is no reason to shift, hence no justification» for its placement at #27 rather than at #26 or at #28 (*Ibid*.: 189). The location of the shift is «arbitrary» (*Ibid.*; 1994b: 65). The difference between contiguous members of the series is so small that any categorial difference between them must appear as «arbitrary». That is the nature of the *soritical series* (1994b: 71). So, a person who sets the borderline at a specific spot cannot thereby be mistaken. If there is a boundary somewhere, it is not the sort of boundary to be worried about.

4b.- Criticism

According to Raffman, one methodological requirement for any adequate solution to the sorites paradox is that it explain why the change occurs from one opposite to the other by means of a soritical series (1994b: 56). However, we have just seen that she herself is unable to provide any account of the reason why the transition occurs at a particular spot rather than at another. For her, the change is a «brute» fact (*Ibid*.: 65), something «mysterious» that simply happens (*Ibid*.: 53). To the extent that, in the latter *loc. cit.*, she asserts that we do not have access to what triggers the change, her whole theory does not meet her own methodological demand. Of course, she has devised an account about how a subject may manage to make a complete assignment to the members of the soritical series, in such a way that the assignment respects the «blurred boundaries of application» of the expressions involved (*Ibid*.: 41); but she has offered no hint as to the mechanism behind the category shift.

Moreover, it is not completely clear that Raffman succeeds in avoiding sharp boundaries. For, with respect to Internal Context 1, patch #26 is red, while #27 has no collor. She thinks that only 'red' and 'orange' are incompatible predicates, but not 'red' and 'no collor', since the latter is merely the absence of any collor (1996: 184, 189). I believe that to ascribe, relative to IC1, a 'no collor' status to patch #27 is to violate the similarity of contiguous members of the series. To affirm that #26 is red, whereas #27 has no status with respect to the same mental anchor does constitute a break of continuity in the sequence. Two adjacent members are accorded a different treatment, with respect to the same context. That is unfair.

Again, something that I cannot fail to mention is that Raffman's conception of borderline cases is apparently not hostile to a paraconsistent conception. She says that patch #26 can be looked as red or as orange; that 'look red' and 'look orange' are very much alike (1994b: 53). Typical contextualists are prone to emphasize the contradictory nature of the sorites, because it is rendered innocuous: they try to dispel the appearance of contradiction by relativizing each opposite predication to distinct respects. We have just seen how Raffman does precisely this. Another case in point is Linda Burns [1991], who, on the one hand, admits that inconsistency is a characteristic of vague language (44), but, on the other hand, she says that vagueness is context dependent: each conjunct of the contradiction, p and not p, has application in a different context (61). According to her, then, we do not follow inconsistent rules (135). In the final analysis, this strategy eliminates the possibility of making simple predications, without qualification; all attribution is relative to a context. An object looks red or orange depending on the background (Goldstein 1988: 452). To say that John is tall is to say that he is significantly taller than what is typical (Graff 2002c: 64). 'x is *F*' means that the degree of *F*ness of x is at least as great as the standard of *F* (Kennedy 1999a: 66). Thus, the strategy of avoiding contradictions by relativizing the application of the predicate to a point of reference bears resemblance to the Aristotelian reduction of the positive degree of an adjective to the comparative degree: things as such are not great or small; they are so by comparison only (*Cat.* 5b, in Moline: 406-7). Yet, for a paraconsistentist this relativization is not sufficiently motivated.

CHAPTER 6

CONTRADICTORIAL GRADUALISM VS. DISCONTINUISM

In this penultimate chapter, I first endeavour to argue that those positions relinquishing the major premise of the sorites have troubles at the moment of explaining the transition question. Such problems disappear if one admits degrees and contradictions. Then, contradictorial gradualism is developed in detail. The comparison will hopefully reveal the virtues of the non classical approach.

1.- The Soritical Series Again

Let me begin by delineating in some detail the notion of a soritical series, which is going to be the basis for my proposed definition of what fuzziness is.

The easy cases of a soritical series are those that are closed on both ends. They satisfy the following three definitional characteristics. Imagine 101 ordered individuals –the odd number is chosen for the convenience of having a midpoint element– such that: (a) the first object, a_0, instantiates a property F clearly, definitely or to the maximum degree; (b) the last object, a_{100}, is not F at all; and (c) any two consecutive items, a_i and a_{i+1}, are very much alike in the relevant respects that it is not the case that only the first is F whereas the second is not:

(CP) $\sim(Fa_i \wedge \sim Fa_{i+1})$.

Let us call this wording of property (c) 'the *Continuation Principle*'. It says that every object subsequent to a_0 is ever so slightly less F, or more not F than the preceding one that it cannot be that a_i is F without a_{i+1} also being F. As an example, take the tallest and the shortest persons in the world now living; and suppose that the difference from one individual to the next in the series is of one millimetre. What (CP) tells us is that it does not happen that only a_i is tall while a_{i+1} is not tall.

The third peculiarity of the soritical collection can be given alternative, though not equivalent formulations. By using logical transformations, from (CP) one can deduce the

(Par.P) *Parity Principle*: $\sim Fa_i \vee Fa_{i+1}$,

but not the

(SP) *Similarity Principle*: $Fa_i \wedge Fa_{i+1} \vee. \sim Fa_i \wedge \sim Fa_{i+1}$.

(SP) asserts that any two adjacent members, a_i and a_{i+1}, are such that, due to their very close resemblance, they should be co-classified: either both are F or both are not F. However, from (SP), the other two principles follow –in *Aj*–, and in this sense (SP) is stronger than (CP). In virtue of these various non interchangeable ways of rendering the third trait (c), the description of the soritical series can accordingly adopt several distinct forms. Defining the soritical series in terms of (SP) is stronger than defining it merely in terms of (CP). Thus, we have opted for a weak version.

Notice further that in the formulation of the three principles above, weak negation is used. So interpreted, they are valid. Nonetheless, if we employed strong negation to convey (Par.P), the definition of the classical material conditional would yield the

(Pre.P) *Preservation Principle*: $Fa_i \supset Fa_{i+1}$, which has an entirely false instance, namely, $Fa_{99} \supset Fa_{100}$. Indeed, it will be argued in section 6 that the penultimate object in the series, i.e., a_{99}, is F to degree 0.01, and that therefore, 'Fa_{99}' is true to degree 0.01; and bearing in mind that 'Fa_{100}' is completely false, it results that $Fa_{99} \supset Fa_{100}$ has a true antecedent but a totally false consequent. For this reason, I will try to keep (Pre.P) out of discussion, as far as possible. And similar considerations apply to (CP) and (SP), both of which are invalid when '$\sim$' is replaced by '$\neg$', as can be easily checked. The convenience of having (CP), (SP) and (Par.P) then is a further motivation inviting the distinction between two negations. On the other hand, (Par.p) will not play a protagonist role either.

If these three principles are true when they are formulated with weak negation, whereas they turn out completely false when conveyed with strong negation, then Classical Logic lacks the resources needed to correctly express what the soritical series consists in. This is a serious drawback. Thus, we have to make a choice. Were we to use CL to state the conditions of the soritical series, we would have a tendency to reject such a putative object because it would be constituted by unsound principles. But, if there is nothing wrong in the soritical series itself, then in order to appropriately render it, we should go beyond CL and appeal to weak negation. So, I again ask the reader to allow me the possibility of differentiating the two negations, to see where it leads.

Summarizing, the easy cases require that the series have two extreme objects, a_0 and a_{100}, which are perfect examples, or paradigms of F and not F, respectively, and such that, any pair of contiguous members, due to their minute variance, must conform with (CP), or (SP). What you should keep in mind is that both (CP) and (SP) hold because the degree of change between a_i and a_{i+1} is as small as you please; in what terms you capture this is secondary.

Notice that a soritical series can be constructed for any pair of opposite properties that are linked to a quantitative variation of an underlying dimension, which, was symbolized by 'G'. Without being rigorous, let me say that it is the numerical fluctuation of this base property G that induces changes in the supervening pair of contrary qualities. For example, short and tall as applied to humans supervene on the height of persons; the condition of being bald or hirsute hinges on how many hairs a man has; cold and hot are a function of temperature, which varies by degrees; whether a person is rich or poor is determined by how much money she has; colours are individuated by the length of the light wave that engenders them, and so on. To be more precise, beside the two opposites, there is a third encompassing property, G, which orders the individuals in the series, and whose increase or decrease causes them to be closer or farther away from one of the superlative members, a_0 or a_{100}.

Two caveats are in order here. First, (CP) or (SP) should not be confused with tolerance, in the exact sense given by Crispin Wright (1975: 334). Remember that a predicate F is *tolerant* when a tiny difference in the possession of the underlying G by two objects does not affect the justice with which F is applied to both. However, it seems to me that tolerance is an antigradualist notion. Fuzziness has nothing to do with tolerance. (SP) by no means is committed to affirming that the degree to which F applies to a_i and a_{i+1} is the same. How much they differ in the possession of F depends on the amount of difference in their possession of G. Different degrees of sharing a property are unimportant only in CL, or classical set theory, but not in a many-valued logic or in fuzzy set theory.

Second, for cases where the soritical series is open on one or both sides, i.e., where there is no element which maximally exemplifies the property, either F or not F, the qualifications 'to the maximum degree' and 'at all' figuring in the characteristics (a) and (b), respectively, should be dropped. And in these cases, it is not evident that (Pre.p) fails.

Is there any soritical series? Well, this is part of the debate to which we now turn. I submit that, if any fuzzy property is real, there must be such a series. For the sake of discussion, I ask the reader to take it as our point of departure. We will later evaluate whether there are good reasons to be offered for its dismissal.

2.- The Nature and the Cause of the Transition

Mark Sainsbury (1992: 179, 186) has investigated a similar problem. He takes one element of the series and asks what the status of its successor is. But our interest is broader. The specific aspect of the problem I want to focus on is how, in a soritical series, the transition from one extreme to the other is effected, if at all. Perhaps this is just another dress the sorites paradox puts on. Yet, it is hopefully a novel, and refreshing look. In approaching the field from this perspective, I deviate a little from the beaten path, and so I attempt to make more visible some partially hidden or neglected spots.

We are going to concentrate on two related questions. Our starting point is the occurrence of the soritical transition from one pole to the other.

Question 1.- First we ask whether the transition from a_0 to a_{100} is gradual, little by little, smooth and continuous, by degrees, or abrupt, sharp, clear-cut, by some sort of jump, or hard line drawing (Cooper, p. 261; Barnes 1982: 53). How does the transition proceed? This is a question concerning the nature of the transition. Here the two obvious alternatives are continuism versus discontinuism.

Question 2.- Why does the transition happen? Our second task consists in explaining what the mechanism or the cause of the change from *F* to not *F* is. What is its condition of possibility?

One should keep in mind that the previous two queries are of such a fundamental importance that every theory of fuzziness has a strong obligation to address them. Failure to provide a satisfactory account of them will be taken as a very serious flaw of any proposal. Indeed, offering a convincing explanation of these two questions will be the test of adequacy for any theory.

3.- Is the Transition Possible?

The problem is that, *given that* any contiguous members a_i and a_{i+1} of the series comply with (CP), this principle seems to prohibit the emergence of a dividing line between them. For a moment think of what would happen were we to employ (Pre.p) instead. This principle, if true –together with *modus ponens*–, would compel us to carry on the application of the predicate until a_{100}, which, by hypothesis, in no way is *F*. The usual sorites paradox appears. Then, we apparently never traverse the boundary from *F* towards not *F*, but always remain within the confines of *F*.

The problem expressed now in terms of (SP) is that we want to know how it could be possible to pass from a_0 to a_{100} through pairs of objects that are distinct from each other so minutely that either both are *F* or both are not *F* (Raffman 1994b: 43, 48). In effect, the soritical series appears at first sight to challenge the possibility of a transition between the opposites, for, if (SP) holds, then, comparing the members of the series pairwise, that is, a_0 with a_1, a_1 with a_2, and so on, we are going to extend the application of the predicate *F* only until some point, say a_{50}, because –as we have just seen– we cannot spread it up to a_{100}. So, suppose the shift takes place at the midpoint a_{50}. But, this means that a_{50} is contradictory, since, compared to a_{49}, it is *F*, while, compared to a_{51}, it is not *F*. Then a_{50} is *F* and not *F*! In a nutshell, if there is a transition, we arrive at a contradiction. And if this is going to be avoided at all costs, then it constitutes *prima facie* evidence for the incompatibility between the soritical series and a transition among the opposites.

If we were employing CL, and more specifically, by *reductio ad absurdum*, at least one of our presuppositions must be given up. The premises were: that a modification has happened somewhere by means of a soritical series, i.e., that a_0 is one hundred per cent *F*, that a_{100} is not a bit F, and one of the three principles discerned above, specially either (CP) or (SP). Which presupposition should be relinquished?

4.- The Nihilist Answer

It is well known that nihilists, like Peter Unger, Samuel Wheeler and Mark Heller, give up the assumption that the predicate is not everywhere instantiated; that is, they reject that there are things which lack F. For them, even a_{100} is F, which shows that everything falls under the extension of F. But, as it is also assumed by the nihilist, the second characteristic of the soritical series postulated that there was an object, a_{100}, which did not fall under the extension of F at all. Then, since we have arrived at an absurd result, the property F is non existent. Therefore, there are no fuzzy properties such as being tall, bald, rich, red, cold, etc.

What to say about this desperate stance? We should evaluate nihilism according to whether it acceptably answers our two questions. In this regard, it seems that most nihilists refuse one presupposition of our inquiry: they believe that there is no transition! For example, Wheeler (1979: 165) affirms that no person can become tall by continuous growth. And in the process of gradual removal of atoms from a stone or a table, Unger (1979b: 136, 132) explicitly denies that there is a change from a stone to nothing, for there was no stone to begin with. Heller in (1990: 79) appears to be of the same opinion, since to the question of at what point the table goes out of existence, he answers that at no point because there was never a table. So, the rejection of transitions constitutes an ingredient of the nihilist stand.

How good is this answer? Well, our purpose was to understand whether the transition was abrupt or gradual, and why it came about. We get no positive clarification, no constructive account from nihilism, because it claims the presupposition of the existence of the transition has been reduced to the absurdum. More than a solution to our puzzles, we are presented with a dissolution of them. Having not responded to our inquiries, nihilism is of no avail whatsoever.

Moreover, when we entertain the extension of CL in favour of a contradictorial gradualist system, a more solid reply to the initial case can be forged: the radical nihilist response is not necessary since a paraconsistent logic can tolerate contradictions –like the one of a_{50} being F and not F– without trivialization. When the negation involved is weak, *reductio ad absurdum* is not logically valid. The mere fact that a set of premises, Γ, entails a contradiction is no reason at all to trigger a revision of the commitments one has made. Then, the following rule is abandoned in a paraconsistent setting: $\Gamma \supset (p \wedge \sim p) \vdash \sim\Gamma$. Note however that, if we replace the weak negation by the strong one, then the inference is perfectly valid. The same criticism will be levelled against discontinuism in the next section.

5.- The Discontinuist Proposal

5a.- Abrupt Transition

Now, let us examine what alternatives are open when one's main motivation is to keep the classical semantics. Remember that we are apparently faced with the choice of sacrificing one of the following: (CP), or the existence of a transition by means of a soritical series. Now, if we admit that there are limits to the application of a fuzzy predicate, we thereby acknowledge that there is a transition somewhere. So, it is (CP) that should go. Yet, if this were so, there would not be any soritical series. The classicist may allege that what the reasoning presented in section 3 reveals is that the three constituents of a soritical series are incoherent, and, consequently, the existence of the series, that we have been taking for granted for the sake of the discussion, is impossible. Hence (CP) would be downright false. There must be an item in the series, a_i, such that it is F, but that its next neighbour, a_{i+1}, is by no means F. Notably among the adversaries, agnosticists (Sorensen, Williamson) and supervaluationists alike (Fine, Keefe) have espoused this viewpoint. They differ in that the former uphold a unique unknowable turning point, while for the latter there are several equally legitimate candidates.

However, we can obliterate this minor disagreement inasmuch as these two trends are united in supporting what will be called the '*Discontinuity Thesis*':

(DT) $\exists a_i (Fa_i \wedge \neg Fa_{i+1})$

Heed the use of '$\neg$' in (DT) as against '$\sim$' in (CP). Therefore, the rejection of (CP) entails –by double negation– (DT): there exists a sharp cutoff point in the series, one marking a neat border between the extensions of the opposites. If we imagine the members of a soritical series placed in a horizontal line, then everything to the left of a_i is F, and everything thereafter is not F at all. More clearly, a_i is the last F, whereas a_{i+1} is the first not F. Thus, we have here a binary partition. (DT) implies that the series is bipartitioned by a_i.

In what sense (DT) has to be contested by a continuist remains to be seen, since one of its substitutions is true, namely, a_{99} is F, while a_{100} is completely not F. It is easy to see that, on the classical understanding of the matter, the intended sense of (DT) is that its left conjunct, in our case a_{99}, does not possess the opposite of F to any extent; otherwise said, a_i is purely F, without any mixture of not F. a_{99} would not differ from a_0 in the having of F; on this respect, they would be on a par. But comparing a_{99} and a_{100} they would have nothing in common; one falls in the extension of the predicate, the other, in the antiextension. Now, thus clarified, (DT) cannot be accepted by a continuist. In order to make the debate more conspicuous, it is perhaps desirable to add the functor of complete truth, 'H', to affect the first conjunct of (DT). For the continuist, the real meaning of (DT) is:

(DT*) $\exists a_i (\mathrm{H}Fa_i \wedge \neg Fa_{i+1})$.

Let us label '*Discontinuism*' the point of view embracing this principle.

Evidently, discontinuism constitutes a way out of the inconsistency, but at the price of losing the most direct way of characterizing the soritical series. This loss is grave and regrettable.

On the other hand, whether discontinuism is a plausible solution to our transition problem depends on its providing or not a satisfactory explanation of it. Has it succeeded in doing so? I have serious doubts.

What is clear is that, concerning our Question 1 of section 2, discontinuism must hold that the change from F to not F comes all of a sudden, without being anticipated or prepared by preceding minor alterations in the possession of F. It is as if the transition were effected in the "span" of a single point. The reason for this is that the series is not tripartitioned, but bisected: a_i draws the line bipartitioning the set into two disjoint subsets. Ontologically speaking, it follows that there is no intermediate situation, no penumbra: *tertium non datur.* Everything would be only on one side of the boundary, but nothing in the borderline. Rather, if the series is bipartitioned, there are no genuine borderline cases, partly F and partly not F, because what is intermediate would be either contradictory or indeterminate, and both possibilities are excluded. There is no in-between ontic status, though from an epistemic point of view, there may be such cases. But these are of no concern to us. Even agnosticists themselves will agree that, at the ontological level, fuzziness obeys the principle of excluded middle, and that any appearance to the contrary, any unclear case, is to be explained away in terms of our ignorance. Therefore, if (DT*) were true, there could not be intermediate situations.

That the transition would be abrupt can be better appreciated by the following illustration, borrowed from David Sanford (1976: 197). A patient is gravely ill by Tuesday, but still alive; by Friday, she is dead. If (DT*) were true, death would be instantaneous, it would not take place during one hour, or a minute, but it would be a matter of an instant –a point of time without duration– since, for any moment of time, it is true that the patient is alive or dead, supposing one is the negation of the other. If dying were limited to the exhalation of the last breath, then, when the person expires, there would be an instant of time, t_i, such that, before t_i, she is still alive –as much as ever–, and there is air in her lungs, however little, while, after t_i, she is already dead. Further, it could not be the case that the patient is partially dead and partially alive when she has breathed out just half of her last breath. Death –on the discontinuist assumption– arrives as soon as the person has exhaled all her breath out. The change occurs not in a stretch but in a point. Still another problem is that one cannot understand how this happens. As we will see more in detail in the next section about the supervenience of fuzziness, the patient's worsening health condition would not proportionally affect her living status in a manner that is reflected in the semantics. If her vital functions gradually decrease so that death seems more imminent, we could not properly say that she is in the throes of death, that she is in transit towards death, with one foot in the tomb. It is excluded that passing away consist in the crossing over a bridge from life to death. Dying, or whatever change in general, instead of being an uninterrupted transition or chain of events, is reduced to a precipitous replacement of two stages, between which there is no *tertium quid*. Dying is punctual; there is no interregnum. Then, death, being sudden and instantaneous, strictly speaking, would cease to be a continuous process.

Ultimately, the source of this way of thinking is a dualistic, or dichotomic conception of reality. Yet, this is inadmissible for a defender of continuity. There are proper borderline cases. Let us imagine Graham walking out of a room at the moment when he is going across the door, and suppose that the point containing the centre of gravity of his body is on the line crossing the centre of gravity of the door frame. So, half of his body is in and the other is not. Now, in general, it may be uncontroversial that an entity must occupy those places where its parts are. Thus, Graham is partially inside and partially outside, since part of his body is in, and part out. Therefore, he is and is not in. This is indeed contradictory (Priest 1998b: 415). But I will claim that precisely this is the nature of all transitions, even of those that seem sharp. Again, we should not be horrified by a simple contradiction, for it is not an over-contradiction. From a paraconsistent perspective, the discontinuist's motivation for her position loses all its appeal. The fact that there is a transition from one opposite to the other, plus the acceptance of (CP) forced on us the recognition of a contradiction, that a_{50} is both F and not F. And this triggered the abandonment of (CP) by those of a classical conviction, which in turn led to (DT*). Yet the argument presented in § 3 is sound and does not constitute any trouble for a contradictorialist, though it is a destructive one for a classicist! As Nicholas Rescher (2001: 9) advocates, accepting a contradiction is a possible reaction to a paradoxical set of commitments.

Moreover, it is only within the framework of CL that we can argue from the existence of a transition to its abruptness. The mere fact that there is a passage from Fa_i to $\sim Fa_{i+1}$, or from a_0 to a_{100}, is not yet a proof that the transition is abrupt rather than gradual (Burnyeat: 336). Even if a precipitating change is very swift, it still must take place by degrees (Rayme Engel: 37). If our frame of reference is a many-valued logic, then we can have a gradual transformation, which exemplifies more of a transition than an abrupt one. See § 6f below for more on gradualist transition.

We conclude that the discontinuity thesis, (DT*), has unacceptable consequences. It does not give us a suitable picture of the nature of the transition.

5b.- Unaccounted Change

On the other hand, we still have to assess how well discontinuism fares with our Question 2. And here things do not appear to get any better. If the transition is not gradual but sharp, by jumps, why does it happen as it does?

Timothy Williamson (1994b: 204) offers an answer when he claims that the meaning of fuzzy terms supervenes on exact facts (and social use, which we are going to leave aside). Remember that we introduced the symbol '*G*' to designate the supervenience base. For example, let us assume for the sake of simplicity that baldness supervenes solely on the number of hairs a person has, independently of its distribution, area covered with hair, etc. According to Williamson, there must be an unknowable quantity a_{i+1} which is the minimum number of hairs a person can have without being bald. So, it comes as no surprise that to the question of how a hairy person can become bald, the discontinuity supporter answers that it is the loss of hair a_{i+1} that makes the difference!

But this sort of position has been looked upon with suspicion since antiquity. Galen, a Greek doctor in the second century AD, criticized it harsh terms: «I know of nothing worse and more absurd than that the being and non-being of a heap is determined by a grain of corn» (Keefe and Smith: 59). What is really queer about this proposal is that alterations in the basic underlying property *G* do not have proportional influence in the supervening pair of opposite properties. That is, if the discontinuity response were true, then a decrease in the number of hairs would not correspondingly affect the hairy condition of the scalp of a person as long as the boundary a_{i+1} is not surpassed. Again, provided we do not exceed the dividing line for 'tall', wherever it may be placed, a person could augment her height remaining always short! This means that the only modification in *G* that produces any transformation in *F* at all is the one involving the cutoff point, from the specific a_i to a_{i+1}, while the rest of fluctuations in *G* would be virtually irrelevant, completely ineffective. The loss of any hair different from a_{i+1} does not make the person bald. Although a person lost hundreds or thousands of hairs, it would not be that, thereby, the person is becoming balder; rather she will continue to be equally hairy. So, again there would not be such a protracted event as being in the course of becoming bald. The person suddenly would become bald the moment she loses hair number a_{i+1}. The transformation would not occur before nor after that particular point. For the view under consideration, changes are punctual.

Yet, this is surely a strange notion of supervenience, having inadmissible consequences. We believe that every difference in the measure of *G* must have an impact on the extent to which *F* is possessed. This proportional correlation between *G* and *F* is not captured by a discontinuist position. *G* and *F* should go hand in hand. The more money a person has, the richer she is; the less height the person has, the shorter she is. It is no objection to say that a person can gain height without thereby becoming taller, or that to be taller does not entail or imply to be tall, because an object x cannot have a property *F* in a greater degree if x does not possess *F* in any degree. How could x be more or less *F* if it is not *F* at all? Only what is *F* can be more *F*. I am not claiming that the classicist is not allowed to make comparisons; my point is that she cannot uphold the general validity of these blatant platitudes, exemplified by 'the less hair a person has, the balder she is'. But the general correlation between *F* and *G* ought not to be restricted by scruples of any sort. There is a lack of proportionality between continuous input *G* and bivalent output *F*/not-*F*. Instead, modifications in *F*/not-*F* should follow in the footsteps of those of gradual *G*.

A worse result of separating the correlative alterations of *G* and *F* is that a small variation in *G* could cause a radical mutation from *F* to its contrary. Nicholas Smith (2004: 166) rightly complains about this. If *G* changes little by little, then *F* too does so. A sudden switch in *F* is explained only by a corresponding sudden switch in *G*; and to the contrary, in the absence of a dramatic change in *G*, a drastic transition from *F* to not-*F* is not accounted for. But in any case, even if the change is abrupt, it must occur through intermediate stages.

A transformation occurs at some point because it was being developed before. But it seems that in the discontinuist framework, birth is not coordinated with the period of pregnancy. Indeed, if a switch from *F* to not *F* occurs somewhere, it would have to be stipulated in a manner that will be artificial, or by mere convention. But no reasonable justification could be offered for the shift; there will be a lack of any principled ground that could plausibly account for the crisp change, as has overtly been acknowledged by Laurence Goldstein (2000: 173) and Diana Raffman (1994b: 53). The former affirms that, when a subject is asked to judge Collor patches in a soritical series, it is an empirical fact that, at some step, she switches her judgement, for no reason, from one object to the next. And the latter asserts that what triggers the judgement shift is something we do not have access to. That this enigmatic mutation is so unnatural is not going to be remedied by resorting to the underlying property *G*, for, if the supervenience relation between *G* and *F*/not-*F* is discontinuist, then we still are deprived of any intelligible explanation as to why the change happens. Suppose that the transformation takes place when the increase in *G* reaches point a_i. But why does it occur exactly at a_i and not at a_{i+1} or at a_{i-1}? This remains a mystery, or it is stipulated by an arbitrary *fiat*. If the basic property *G* is to discharge its explanatory role, then the connection established should take a gradualist form: the more *G*, the less *F*, or the more *G*, the more *F*. A proportional correlation –either direct or inverse– among *G* and *F* is far more illuminating than a discontinuist one.

In this regard, it is instructive to contrast two notions of supervenience. Timothy Williamson, in 1994 (1994b: 203), defines supervenience sharply; simplifying, he says that:

> if x has «exactly the same» measure of *G* than y, then x is *F* iff y is *F*.

Thus, one can attribute baldness to two individuals, A and B, depending on whether they have identical number of hairs, neither one more, nor one less. When this condition is not met, or more exactly, whenever each of them falls on a different side of the border line, for example, when A has 49,999 but B has 50,000 hairs, it may well be the case that A is bald whereas B is not, assuming that the point 50,000 bisects the series. So formulated, discontinuist supervenience then delivers (DT^*). Years later, Williamson (2002c: 53) has availed himself of a gradualist version; he says:

> if x is similar enough to y, and x is known to be *F*, then y is *F*.

Here the condition is relaxed. But, if degrees do play a role in the determination of a property, then they should be an essential part o the explanation, making us reluctant to accept any sharp cutoff. Compare this second formulation with another quantitative principle given by Myles Burnyeat (238), here simplified:

> if x deserves treatment *F*, and y does not differ significantly from x in *G*,
> then y deserves *F*.

Note how this is congenial with (CP): *F* can still be applied to a_{i+1}, only marginally differing from a_i, to which *F* has been applied. Now, *F* continues to be attributed in spite of a small deviation in the extent of *G*; but of course, the degree to which *F* is attributed must also diminish or rise by a similar margin.

Thus, we arrive at the conclusion that a drastic change from one opposite to the other has not been explained by an insignificant loss of a single hair or by the removal of one grain. The direct proportionality among *G* and *F* should not be sacrificed; rather, any theory should be supple enough to accommodate it. Because of its rejection of degrees, discontinuism does not convincingly explain the transition.

6.- Contradictorial Gradualism

I have argued so far against nihilism and discontinuism; now I will make a case for my own point of view: contradictorial gradualism.

6a.- Fuzziness

What is fuzziness? From the point of view here advocated, we can conceive of fuzziness as the phenomenon which manifests itself in the intermediate zone of a soritical series. Let me characterize this anew. First, at least for the easiest cases, the series is closed on both ends, in the sense that there are elements maximally exemplifying both opposites. They are the extremes, say, a_0 and a_{100}. And second, between the two poles there is the fuzzy area (Horwich 2000a: 88). Fuzzy situations, or borderline cases, are all those in between the extremes, from a_1 to a_{99}, both included. This region consists of the overlap of *F* and not *F*

(Rescher 2001: 77; Black 1937: 39; Cooper: 260). This mid zone nowhere is homogeneous, but admits of different percentages (in Read 2003: 6) of mixture in such a way that, as the blending becomes less *F*, it gets more not-*F*. Thus, a_0 is 100% *F* and 0% not *F*. Its next neighbour, a_1, is 99% *F* and 1% not *F*. a_2 is 98% *F* and 2% not *F*, and so on. In general, there are no two consecutive members to which *F* is attributed in the same degree, because they differently instantiate the underlying property *G*. From object to object there is a tiny, minuscule difference, imperceptible, but not negligible (McGee and McLaughlin, 220). a_{50} occupies a unique position in the series, being the only one symmetrically placed among the opposites; of no other point can we say that it equally exemplifies *F* and not *F*.

Then, the two features of fuzziness that should serve as earmarks are its being nothing but gradual *and* contradictory (Godard-Wendling: 2427; Dubois, Ostasiewicz & Prade: 34; Kosko: 46, 85, 155). In the subsequent sections, we elaborate on these two aspects.

This recent characterization of the soritical series introduces two novelties with respect to the one previously given in § 1 above. Firstly, I have not made appeal to any notion of similarity (SP) nor of continuation (CP), that play a key role in the generation of the sorites paradox. The current definition can in this manner be assessed in itself, apart from any issue arising from the paradox. Secondly, the contradictorial nature of the series has now been built in directly, without the mediation of any further principle.

It may be objected that the second condition of the soritical series is to blame for the genesis of incoherent situations. But this is not the case without the concourse of further auxiliary principles and CL. The soritical series as such is not only possible, or feasible, but actual and real. Of course, this does not mean that it is not contradictory, for it is indeed so, but it is not absurd. Being simply contradictory is not the same as being impossible. What may be wrong is the logic that allows you to conclude that a_{100} is *F*. This is indeed a *non sequitur*.

Concerning the ontological question of whether fuzziness is a feature of reality, we can logically say that the world itself is fuzzy in that it contains fuzzy facts, which consist of –in the case of the monadic ones– an object possessing and / or lacking a property to a limited extent. A fuzzy property *F* is just one which can be exemplified to different degrees, from maximal to minimal, passing through all intermediate stages. Fuzziness is a real phenomenon. Reality itself is gradual.

It should have been observed that the fuzzy zone is precisely delimited by the extremes of the series -if there are any. Does it mean that, in these cases, there is no higher order fuzziness? In a sense, higher order fuzziness is inexistent, since there is no indeterminacy or uncertainty concerning which cases are to be taken as borderline. But in another sense, there is a second order fuzziness, because the question of how fuzzy a member of the series is admits of a gradual answer. a_{50} is the fuzziest case, and those elements which are closer to it are fuzzier than those which are closer to the extremes; the latter, therefore, are much less fuzzy. Fuzziness itself thus comes in degrees.

On the other hand, in the cases where instead of having a member exemplifying the property *F* to the maximum degree, we have an unending series of elements everyone of which instantiates *F* to a lesser or greater degree, we humans have no way of determining the exact degree of possession of the property by any individual in the series, since we lack a point of reference with respect to which we could make a measurement. If we had a fixed paradigm, we could assign to every member a particular position within the scale. But in the absence of a standard, we are at a loss. In cases like these, we have a common ground with agnosticists. But there is a difference. For us, the ignorance is only human, not of principle. An omniscient deity -if there is any- would know the degree of possession. How? If she is also almighty, she does not need to apply any procedure to have access to truths. In any case, the problem of which degree an object possesses is epistemological. But my claim is ontological: it is a determinate matter of fact whether an object has a property or not, and if it does, to what degree. In unbounded series, we do not know which this specific degree is. The point was elaborated in section 6 of Chapter 4.

6b.- Degrees of Properties

Several authors, and not only those supporting the many-valued or fuzzy approach, have acknowledged that in a soritical series, the variation is gradual (Sainsbury & Williamson: 475; Wright 2003c: 91; Horwich 2000a: 83; Leibniz' 1676 *Pacidius Philalethi* (in Levey); Hospers: 120; Edgington 2001: 375; Pascal Engel: 534; Dubois, Ostasiewicz, and Prade: 27; Walton: 209, 57; Cook [2005]). And with reason: the core feature of fuzziness is its graduality: more-or-lessness (Sylvan & Hyde: 26), difference by small degrees (Labov: 353).

In support of this, we put forward two considerations.

First, there is well known textual evidence that ancient authors were fully aware of the pivotal role that degrees played in our subject. In antiquity, fuzziness *per se* was not an independent, self-standing topic of discussion, but was touched upon within the context of the sorites, or heap paradox, which was also called the Little-by-Little argument, ὁ παρὰ μικρὸν λόγος (Mignucci: 232), one that proceeds by small transitions (Burnyeat: 318). Cicero explains that the reasoning develops «by minute steps of gradual addition or withdrawal» (in Leib: 149, n. 2). The typical questioning was: Is one few or many? Is two few?, and so on. Galen defines a heap in the following way: «besides being single particles in juxtaposition, it has quantity and mass of considerable size». Again, he illustrates the soritical questioning: is a single grain a heap? Are two grains a heap? ... And he continued asking whether «the quantity of each single one of these numbers constitutes a heap». The procedure was «gradual addition of more» grains (in Keefe & Smith: 58-9).

Cicero also affirms that «the nature of things has provided us with no knowledge of boundaries... if we are questioned by degrees» (in Barnes 1982: 34), or –according to other translations– «little by little» (in Burnyeat: 325), «by gradual progression» (in Keefe & Smith: 60). He objects to the stoics that their theory «does not teach what is the lower or upper limit of increase or decrease» (*Ibid.*). Galen additionally says that if the sophism were valid, it would prove the inexistence of anything having «a measure of extent», like a mountain, a crowd, a city, etc. He wants to inquire whether «there is in the nature of things some measure of the 'very many times', or whether there cannot in any way at all be a measure...» (Barnes, *Ibid*: 62).

The vocabulary displayed in all these quotations is explicitly quantitative: addition, increase, decrease, measure of extent, etc. Indeed, the word 'sorites' (σωρίτης) means a heaper, or accumulator, the person who adds grain to grain (*Ibid*. 32, n. 18). From these texts we gather that, in the ancient perception of the matter, the puzzle was mainly generated by terms amenable to quantification. How much money do you need to be rich? (Burnyeat: 318, 325-6). How many grains of wheat are required to make a heap? (Brock: 46). Indeed, all the instances of proper soritical series used in antiquity are numerical (Bobzien: 227). The ancient solutions to the riddle may have been sceptical or dogmatic, but the language of the dispute was gradualistic. That many-valued logics and fuzzy set theory have made their appearance just recently should not obscure the fact that, since the onset of the transition problem, graduality was present.

Second, perhaps one of the strongest grounds to postulate degrees is an argument to the best explanation of how gradual change is possible. Just consider what would happen if the degree of possession of *F* by the various objects in the soritical series were the same. If a_n were as *F* as a_{n+1}, then I cannot see how the successive members will stop being *F* in a non arbitrary way. Precisely an *unceasing* property has been defined as one whose extent is preserved undiminished (R. Engel: 37, n. 17). How difficult it is to plausibly account for the transition from a discontinuist point of view was seen in section 5b above. Therefore, if there are continuous transitions, properties must lend themselves to be possessed to varying intensities. Otherwise, to borrow an example from Rayme Engel (28), if rigidity were not gradual, there could not be any stiffening, nor losing or gaining rigidity. If there were no gradual properties, there would not be any smooth change either. This is made possible only by degrees. If there is to be a genuine transition, it must be realized through intermediate stages (Ausín and Peña 2001).

We conclude with Christopher Kennedy that degrees are part of the ontology, and that amounts and measures need to be introduced in the semantics (1999a: 77; 2003a).

Now, if *a* can be *F* to different extents, if *a* can be more *F* or less, then it means that being *F* is gradual, and so being itself comes in degrees, if we do not want to be essentialists, driving a wedge between being-so and being as such. The degree of China's being large is nothing but the degree of existence of the fact that China is large. If the extent of the cruelty of St. Francis of Assisi is small, then his cruelty is almost non existent. In general, if the *F* of *a* has a measure of Δ, then the *F* of *a* is real to the same degree. Therefore, the gradual nature of properties entails an identical gradual nature of existence. Existence is merely another fuzzy property[22].

6c.- Degrees of Truth

To argue for gradual properties is one thing, to argue for degrees of truth is another. In this section, let me say a few words about how we can go from the former to the latter.

Let us begin with the Tarskian requirement that purportedly should be part of any conception of truth. Restricting ourselves to the atomic case, the connection between truth and satisfaction is established by means of the schema:

(RT) '*a* is *F*' is true iff *a* is *F*.

We will call this principle, '*Redundancy Truth*'. The schema has been upheld by deflationary, disquotational, or redundancy theories of truth. Actually, not making any use of the technical meaning of the satisfaction relation, the sense in which (RT) will be understood here is that sentence "p" attributing property *F* to object *a* holds true iff *a* has property *F*, i.e., whenever there is some fact consisting of *a*'s possessing *F*, i.e., iff the real world is as "p" says it is. So, what (RT) lays down is the necessary and sufficient conditions that must obtain in reality to assign the predicate 'true' to a sentence, written or spoken.

Indeed, when we accept a many-valued logic, we can strengthen (RT) by placing a strict equivalence instead of the mere biconditional. Thus, we get a stronger version of (RT), namely

(RT*) That '*a* is *F*' is true is equivalent to *a* is *F*.

Remember that in section 1a of the Introduction, we distinguished the simple conditional, '$\supset$', from the implication, '$\rightarrow$', and correspondingly, the biconditional, '$\equiv$', from the equivalence, '$\leftrightarrow$'. If "p" is equivalent to "q", then both sentences have exactly the same truth value. But "p iff q" can have a designated value even if "p" and "q" have different truth values.

Once we have a genuine equivalence in place, a gradualist version of truth will be built on the basis of (RT*). Regardless of the dispute of whether (RT*) is all there is to truth or whether something else ought to be added, we should acknowledge that (RT*) is neutral with respect to bivalence or multi-valence, in that there is nothing in the formulation of (RT*) that prohibits the introduction of degrees on each side of the schema, nor does it force a binary interpretation. So, (RT*) is a good candidate for a starting point: it is our first premise. But we have seen in previous sections that there are gradual properties; that is, the right equivalent of (RT*) is amenable to fluctuate by degrees. Being *F* is something that can be possessed in a greater or lesser degree. Therefore, by substitution of equivalents, the left equivalent of (RT*) has to be also gradual, truth itself must be a matter of degree. Thus,

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1 See Vásconez and Peña [1996] for a discussion of a gradual ontology.

degrees of truth are an immediate consequence of the graduality in the possession of properties and the redundancy theory of truth.

Otherwise said, the structure of the short argument in favour of degrees of truth is: $p \leftrightarrow q$, ...q... $\vdash$ ...p..., where the blank space in the second premise and in the conclusion stands for a single context wherein one of the sentences occurs. This rule is known as the Replacement of Equivalents. The case began with the identity between the truth of a sentence and the fact it expresses, designates, refers to, represents, affirms, etc. This is (RT*). Then we noted that its right member is susceptible to vary in degrees. And the conclusion that there are degrees of truth was drawn applying the rule of inference mentioned. Anyone unwilling to accept the conclusion must reject one of the premises or, more unlikely, the validity of the argument. The latter option is very hard, since the rule is of much use in logic and in itself unproblematic. And the two premises were the graduality of properties –which is manifested in our way of talking–, and (RT*), which can be traced back to the Material Adequacy Condition or Convention T of Tarski's conception of truth, allegedly a minimal requirement for any realist conception of truth. Both premises have their own backing. So, the conclusion of gradual truth seems justified.

We may now proceed to generalize (RT*), as it is customarily done in many-valued and fuzzy logic: the extent to which the sentence '*a* is *F*' is true is identical to the extent to which property *F* is possessed by object *a*. Hence,

(GRT) That '*a* is *F*' is ... true is equivalent to *a* is ... *F*,

where the two blank spaces should be uniformly filled by a single expression indicating the respective degrees (Grim 1997, § 4). This will be called the *Generalized Redundancy Truth*. "p" is true in exactly the same measure as the fact denoted by sentence "p" is real. For example, to say that it is very true that 'Tartuffe is a hypocritical man' is the same as saying that Tartuffe is a very hypocritical man. The right member of (GRT) can also be couched in terms of the degree of membership of *a* to the extension of the predicate '*F*' (Goguen: 331, 333; Lakoff 1973: 460, 466, 491; Bouchon-Meunier 1995: 100, 117, 120; Gottwald 2001: 25, 424-25; Dubois, Ostasiewicz and Prade: 27; Smets and Magrez: 67. But cfr. Machina 1976: 65, 58, 75). In this case, degrees of membership in a set imply degrees of truth and vice versa. Thus, we need as many degrees of truth as there are degrees of belonging to a set. If we have distinguished one hundred degrees of membership, there will be one hundred corresponding degrees of truth.

Concerning the semantical status of the intermediate degrees of truth, we both designate and antidesignate all of them. The reason for this is that, by (GRT), degrees of truth are to reflect every intermediate stage of change in the soritical series, from totally *F* to complete not-*F*; and we saw, in section 6a, that the whole stretch of borderline cases is contradictory: as *F* recedes, it –in the same measure– makes room for not-*F*. So, all degrees save 1 are false (or antidesignated), and all degrees except 0 are true (or designated). And ½ is the only one which is half true, half false. The next section continues the argumentation in favour of this designation.

That there is an infinity of degrees of truth may be shown by those properties that encompass a non enumerable gamut of degrees of membership, such as the property of being close to the Eiffel Tower, being large, etc. As for the question of why the real numbers are chosen instead of the rational numbers as the set of the degrees of truth, the reason is that they are required in order to have continuity. Ontological *continuity* is understood as demanding that, between any two entities, there must be all the intermediate possible degrees. With the rational numbers, we will have a series of intermediate stages; yet, we would not have a transition but a series of leaps. Density is not sufficient for continuity. The function that raises a number to the power of two is continuous; however, with the rational numbers, we do not have the square roots.

6d.- Minimalism vs. Maximalism

We can approach the issue of which the designated truth values are from another angle, namely, by asking what the lowest level of truth required for a sentence to be true is. Given the identity (GRT), the question amounts to what the minimal threshold of *F* allowed for a thing to qualify as an authentic *F* should be. Remark that our concern is semantical but not pragmatical: we are not inquiring about what amount of truth a sentence should have in order for it to be permissibly asserted in a conversational context. What restrictions must be met by assertions to be properly uttered in a specific situation is a quite different matter, for there are circumstances in which the total truth of a sentence is not enough to authorize its statement. Rather, we are interested in how much truth should be demanded from a sentence to be [rightly judged as simply] true.

By *maximalism* we are understanding the position that countenances the following *Maximalization Rule*:

(MR): "p" is true $\vdash$ "p" is completely true.

And by *minimalism*, I will understand the position that considers "p" true provided that it is not completely false; i.e., whatever sentence having a degree of truth greater than zero is true.

Should we opt for maximalism, and refuse to accept as true any sentence having a value lower than 1? Strange as this may sound, this is not a position concocted to fit the dialectics of the discussion, in need of having an adversary. No, the position has in fact been voiced by some outstanding philosophers who have rejected the existence of degrees of truth, or gradable properties. More than one reader may have wondered whether there had ever been a philosopher who had flatly denied that a property was a matter of degree. Astonishing as it may appear, this opposition to gradual properties is real. Two illustrious defenders of it are Bertrand Russell and Crispin Wright. Here are a couple of quotations. Russell (62) expresses that:

> Nothing is more or less what it is, or to a certain extent possessed of the properties which it possesses.

Wright (1987a: 255) epitomizes this line of thought:

> any vague concept F admits of quite a wide variety of discernible cases all of which are *definitely* and *absolutely* F.

Similarly, Romerales (52) defends that there are different shades of green, but that they all are fully green.

Wright (*Ibid.* 262) also declares that «...there is no apparent way whereby a statement could be true without being definitely so». More examples of alethic maximalism are the following. Leibniz says

> -Can the truth of some proposition increase or decrease... in the same way as water gets hotter or colder by degrees?
> -Certainly not. ...a proposition is either wholly true or wholly false (in Levey).

Frege affirms that, if there can be no complete truth,

> nothing at all would be true; for what is only half true is untrue. Truth does not admit of more or less (in Candlish, Section 1).

Michael Dummett (1970: 256) also joins the choir: «the only possible meaning we could give to the word 'true' is that of 'definitely true'». Timothy Williamson (1994b: 194) asks: «what more could it take for an utterance to be definitely true than just for it to be true?». And Rosanna Keefe (2000: 27) echoes that «no sentence can be true without being determinately true». And what is even more surprising, some advocates of a graduality persuasion have succumbed to the maximalist illusion (Edgington 1996: 299; Gottwald 2001: 425; besides, Goguen and Lakoff).

Though this position has been supported by such eminent minds, it is not correct. I will argue against it supposing Redundancy Truth, (RT). I will try to show that applying maximalism to the possession of properties is far too demanding, for it requires that in order for an object to have a property, it needs to exemplify it to the utmost degree. It would mean that only the person who has 0 hairs is bald, or only the tallest person in the world is tall. If this were so, we will be deprived of all the intermediate cases of a soritical series, and we would be left with just the two extreme poles. The lover of the extremes is content only with the paradigm cases of each property, all other peripheral cases being erased out of the map. The extension of a predicate would be very poor, consisting of only the best exemplars. Perhaps not all saints will be good enough, nor every hero will be brave enough.

How inconvenient maximalism is will be manifest by considering one of its most illustrious incarnations, utilitarian ethics. I will not challenge that perhaps there is a way to make comparative assertions of the goodness of an action, albeit Alistair Norcross (22-23) doubts. My present concern is with the utilitarian notion of obligation. Which action ought we to perform? The standard utilitarian answer is that the right action is the one that maximizes the amount of intrinsic good, one that, among all the possible alternative actions, has the best results for the majority of affected people. This position has the consequence that the other contemplated actions with a lesser amount of realizable good are evaluated as not licit at all; thus the second best alternative comes to be as illicit as the worst one. The classification of actions in respect of right and wrong is dualistic, not gradual. The right action is at the superlative level; all other possible actions are judged as contrary to duty, without differentiation. But it is clear that the utilitarian confuses the right with the optimum, for an action can be morally justified, permissible, or even mandatory without being excellent, as the plenty of counterexamples to utilitarianism have demonstrated. Paraphrasing Michael Stocker (312), we can say that sometimes doing what is best is wrong. Analogously, if what is less than absolutely true can be true enough, maximalism is mistaken.

Perhaps maximalism is backed up by a *reductio ad absurdum*. Consider a series composed of adjacent points, beginning with point A, and ending with point Z, which, by hypothesis, is not in the least close to A. We are interested in knowing which points are close to A. Obviously, B is close, since it is contiguous to A. What about point C? One can argue that, since C is close to B, and B is close to A, then C is also close to A. But if one allows this kind of reasoning, then one embarks upon a slippery slope argument, whose consequence would be that not only C, but also D, E, F... and Z are close to A. But since this last outcome is absurd, the only way to stop going all the way down –so the maximalist could allege– would be to uproot the mistake, by refusing to allow that C is close to A. But this would mean that the sole point close to A is B, all other points being not close [at all]. In other words, the aftermath would be that in order for a point to be near A, it must be so close to A that being closer to A is not possible! Therefore, only that which has the superlative degree of *F* deserves to be named *F*.

However, this is surely an excessive requirement. We can admit that C is also close, but less; D too is near, but still less so, and so on. We can take the slippery slope as proving that everything is close to A, the premise to be reduced to the absurdum being the supposition that point Z is not the bit close to A. If we had to choose between maximalism and the inexistence of a point which is 100% not close to A, the option for the second seems not to be too embarrassing since the notion of a point which is perfectly and totally distant from A does not make sense if space is infinite, as it could be maintained.

Now, this answer may cause some qualms with the maximalist. She could reply that, in those cases where there is no paradigmatic object exemplifying *F* to the utmost degree, there are no *F* objects. (Actually this is how Frege argued in the quotation few paragraphs before in this same section). That is, where the series is open on one side, without there being an object which is *F* to degree 1, everything will be not *F*. For example, where there is nothing that is a complete heap, since an atom can always be added, there are no heaps. And similarly, there are no tall men, no hairy persons, etc. But it may be objected that this is almost as absurd as the nihilist position or trivial, for everything has or lacks the property in question.

This objection is serious but not really troublesome. I accept that, in unbounded series, it is true that nothing is a heap, or that everybody is short. Yet we should ask what the degree of truth of these assertions is. And the answer is that they are minimally true. Take the first case, a universally quantified negation, $\forall x{\sim}Fx$. Here the negation involved is weak. The truth value of this generalization is the *infimum* of the set of truth values of all of its instances: ${\sim}Fx_0$, ${\sim}Fx_1$, ${\sim}Fx_2$, and so on without end. As the number of grains keep increasing, the truth value of the successive sentences in this series diminishes accordingly, and asymptotically approximates zero. Now, in Peña's semantics for the predicate calculus *Aq*, there is one infinitesimal[23] degree of truth, namely, α, which is equal to $1/\infty$. And this is the truth value of the generalized sentence "nothing is a heap". So, if it is infinitesimally true that 'there are no heaps', then its weak negation, that "there are some heaps" is $1\text{-}\alpha$ true, i.e., infinitely true, but less than completely true. Thus, the perspective appealing to degrees is quite different from nihilism, though there is an infinitesimally true interpretation of it from the gradualist point of view. On the other hand, the gradual conception is not trivial either, for no super-contradictions can be derived in the system. To accept both that it is infinitesimally true that there are no heaps and that it is infinitesimally false that there are heaps is not absurd. (Symbolically -using Peña's notation-, one may assert both: $Y{\sim}\exists xp$ and $b\exists xp$). Strictly speaking, it is not even a contradiction, i.e., a formula of the form $p\wedge{\sim}p$. It is not the same as accepting both that there are no heaps at all and that there are heaps, $\neg\exists xp \wedge \exists xp$. This is over-contradictory, but not the previous acceptance.

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[23] The mathematical notion of infinitesimal employed here does not differ from that of Leibniz or of the non-standard analysis of Abraham Robinson, namely: quantity α is an infinitesimal if it is greater than 0, but smaller than all standard positive real numbers (Priestley, 362; Edwards, 264; Peña 1993a: 82; Rosser, 558; Robinson 1967: 539). Another characteristic of an infinitesimal is that it does not satisfy the Archimedes' axiom, which says that from every positive number, z, smaller than 1, we can obtain a number greater than 1 by repeated addition: i.e., $z+z+...+z$ (x times) >1, where x is an ordinary natural number (Robinson 1967: 543). A similar characteristic is acknowledged by Leibniz in his definition of incomparable quantities (See Horváth, p. 63).

The postulation of an infinitesimal is motivated by a desire to avoid ω-superinconsistency (that is, that although all the instances of replacing a denoting sign for a free variable in "p" are truly affirmable, their universal generalization is completely false. See Peña 1991: 125-26, 174-77, 187-88; 1985: 485-95).

Two differences are worthwhile mentioning between Peña's conception and that of Leibniz and Robinson. One is that, in contrast to the latter thinkers, who thought the infinitesimal is an imaginary or fictitious notion, but a useful tool, the former believes that the infinitesimal is a real entity, but one whose existence is only infinitesimal in all respects. The other discrepancy is that, in opposition to Robinson's practice, Peña postulates just a single infinitesimal, instead of an infinity of them. There can be only one infinitesimal because this is understood in a strong sense as an entity having a degree of reality of $1/\infty$ in all its aspects, and by the ontological principle of strict identity, "two" entities having exactly the same degree of existence in all respects are one and the same.

Somebody may think that a third way might be open beside maximalism and minimalism, namely, to fix an intermediate threshold, for example 50% as the minimum measure of truth for a sentence to be true. The problem with this is that it is arbitrary to fix any lowest level different from the one immediately above 0. In the case at hand, why not, for example, to set the limit at 49,999%? We will encounter the problem of which point to pick out to mark the transition.

Well, I hope these considerations lend plausibility to minimalism. We live in a world of imperfect realizations. To ask for nothing less than supreme exemplars is going to leave us almost empty handed. We better get ourselves reconciled with this less than perfect surrounding reality, and accept that what is not completely not-*F* is *F* to some degree.

If (MR) is unpalatable, and intermediate positions unstable, we better opt for granting a designated status to all degrees except 0, which is totally false. Thus, we arrive at the *Endorsement*, or *Acquiescence Rule*: if x is *F* up to a non-zero degree, then x is *F*, *tout court*; in order for a sentence to be true, it suffices that it be true to some extent.

(AR) "p" is more or less true $\vdash$ "p" is true;

x is more or less *F* $\vdash$ x is *F*.

Therefore, we cannot but agree with Graham Priest (2003:16), when he asserts that: «For something to be acceptable, it does not have to have unit truth-value».

That (AR) in its alethic form is a valid rule, can be seen by checking that it conforms to the definition of a valid inference: if its premise is true, then it is impossible that the conclusion be entirely false. In fact, supposing that "p" is true to some degree or other, then what the conclusion declares is that "p" is true, omitting the extent to which it is so. Since "p" is not completely false, by hypothesis, then "p" must receive some designated value, whatever, and hence, be true, for a designated value ascribed to a sentence makes it true. Here a principle of excluded middle is operating: for any "p", either "p" is completely false or else "p" is true (to some degree).

In conclusion, if minimalism has some credibility, then we should take up (AR). In the next section, we will see how (AR) is used to derive a benign contradiction.

6e.- From Degrees to Contradictions

We have seen that whatever is intermediate between two opposites has a share in each of them, partaking partially in the nature of both. We now present an argument to the effect that degrees imply contradictions (Machina 1976: 54-5; Read 1995: 173; Cfr. Pinkal 1995: 159-60). If fuzziness is gradual, it is bound to be also contradictory. This contradictoriality is its second definitional characteristic (Machina 1976: 59; Hyde 1997: 649; Labov: 356; Kosko: 23, 125; K. Lehrer, in Sorensen 1991b: 96).

Let us suppose that object *a* falls short of absolutely exemplifying property *F*. Cases of this sort are abundant: a bus may be full, but not replete; a book is interesting but not too much; a blackboard may be clean enough, and yet not thoroughly clean, etc. Indeed the majority of the objects of our sensible world are deficient instances of properties. Well, let us take one of those innumerable objects. Now, why is it that *a* is not completely *F*? Because *a* is in some measure not *F*. Why is water impure? Because it is mixed with something other than water. What accounts for *a*'s imperfectly exemplifying *F* is its being not *F* to some degree. In general, an object not wholly instantiating *F* has to have a share in the opposite of *F* (Kosko: 85). Inasmuch as the door is not completely closed, it is somewhat open. So, we have a situation in which *a* has *F*, but only partially, and this occurs simultaneously with its possessing not *F* to some extent.

Now, in order to see that this fuzzy case is contradictory we only need to apply (AR) to each conjunct. For if *a* is partially *F*, it is *F*; and if *a* is not *F* to some extent, it is not *F*. Therefore, it is *F* and not *F*.

Note that the contradiction arrived at is possible only because the object possesses both opposite properties in a limited extent. It is the gradual possession which makes this contradiction possible. But in a paraconsistent framework contradictions of this type are completely harmless and innocuous: they can be kept without affecting the health of the system. Indeed, they are an advantageous addition.

6f.- Gradual Transition

How is the transition among the opposites to be described and explained in a gradual ontology? It occurs in the following manner. When we move along the series away from a_0, according to the extent of *G*'s variation, the change makes its inception with element a_1, since this is 99% *F* but 1% not *F*. As we pass across the successive members, the degree of *F* diminishes to the same extent as the degree of not *F* augments. At the moment we reach a_{50}, both dishes of the scales keep a balance between *F* and not *F*. But immediately after we depart from the midpoint and go towards a_{51}, the weight of not *F* makes the scales be tilted towards its own side, the more so, the more we go beyond. a_{51} may rightly be considered as the preeminent turning point because, being 51% not-*F* and 49% *F*, to say of it that it is not-*F* is to say something truer than to say of it that it is *F*. Conversely, saying of it that it is *F* is to say something falser than to say of it that it is not-*F*. Here we follow a principle first enunciated by the old Pre-socratic pluralists: a thing should be named after the element whose presence has the highest proportion. However, that an object should be named after the property which is more predominant should not make us lose sight of the fact that the mixed object contains a share of the other opposite too.

Thus, a_{50} is *F*, but a_{51} is not *F*. Is there here a cutoff point? In a loose sense, we have a limit here, because we pass from *F* to not-*F*. But in another sense, this boundary is soft because a_{50} is also not-*F*, and a_{51} is *F* too. So, both are *F* and both are not-*F*, but not in the same amount, the difference being gradual. The similarity principle, $Fa_i \wedge Fa_{i+1} \vee . \sim Fa_i \wedge \sim Fa_{i+1}$ is preserved, i.e., it is not completely false; in fact, its scope of truth –in the case at hand– ranges from 0.5 to 0.99 true. What about the continuation principle, $\sim(Fa_i \wedge \sim Fa_{i+1})$? It remains true, but it is also false. In the particular instance of $\sim(Fa_{50} \wedge \sim Fa_{51})$, it is as true as false, for "Fa_{50}" is 0.5 true, and "Fa_{51}" is 0.49 true, but then "$\sim Fa_{51}$" is 0.51 true, and the conjunction is 0.5 true, and so too its weak negation. So, (CP) is never falser than 50%; thus, it is truer than false. But as a_i approaches the extremes, (CP) gets closer to be wholly true. And in the case of the last pair considered, a_{99} and a_{100}, it is true to degree 0,99.

We can conclude, there is no discontinuity (Black 1963: 10). The fact that there is a transition does not entail that the transition must be abrupt. On the contrary, it is gradual (Cooper: 261; Hospers: 120; Sadegh-Zadeh: 7). And this is the answer to our first question concerning the nature of the transition.

To end this section, let me make it explicit what our answer to the second question of section 2 is. Why the transition occurs? *F* changes because of proportional change in the underlying property *G.*

7.- Conclusion

After a characterization of the soritical series, I set out to inquire two aspects of the transition question: how does the change from *F* to not *F* happen, and what generates the transition? In section 3, I presented an argument to show that the soritical series was contradictory, and later in sections 4 and 5, we saw that there was no compelling reason to reject the soundness of the reasoning. Discontinuism was revealed to have a conception of change as a precipitous and instantaneous replacement of two stages, and was unable to satisfactorily explain why the transition takes place. On the other hand, accepting degrees and [benign] contradictions makes a smooth transition possible. To do justice to fuzziness, characterized as nothing but gradual and contradictory, we should resort to a many-valued, paraconsistent logic together with fuzzy set theory.

One corollary of this chapter is that, if reality is gradual and contradictory, we had better adjust our logical system to mirror degrees and contradictions. Otherwise, a firm attachment to a bivalent superconsistent framework will only result in a loss of data and impoverishment of reality (Cfr. Besnard and Hunter: 4).

CHAPTER 7

CONCLUSION: FUZZINESS VINDICATED AS GRADUAL AND CONTRADICTORY

The presentation of proposals is over. It is time to recapitulate the main ideas advanced.

As in the majority of serious philosophical problems, we cannot claim to have solved the problems we set up to investigate. What I have done -I think- is just to underpin a particular version of a fuzzy, many-valued, paraconsistent solution to fuzziness and the sorites paradox. Or, perhaps, more realistically, I have at a least made it more difficult to dismiss this sort of approach. I have tried to answer to the most cogent objections to the present view, and exposed my reasons against alternative approaches, which have been presented with their respective motivations. All along, I was keen to clarify the meaning of the terms used.

By far, chapters 1 and 6 are the most important of the dissertation.

Chapter 1 was an attempt to justify the tenet that fuzziness does support the major premise of the sorites.

I recall the fact that the logical systems here used, *Aj* and *Aq*, are a strict extension of classical logic relative to a particular reading of its negation operator, that is, all classical tautologies, theorems and rules of inference were kept in the new system. So, *Aj* and *Aq* are not rivals of CL, but generalize it.

On the other hand, a distinction that deserves being remembered is that between two versions of the *principle of bivalence*. In its strong form, claiming the existence of just two truth values, mutually exclusive and jointly exhaustive, the principle was rejected. However, we kept a weak version of it, that every sentence is either true or false. Indeed, we did not find any reason based on fuzziness to give up this precept.

I devoted sections 1e and 3a to the necessary clarification of our own stance in the background debate between realist and anti-realist conceptions. We contrasted two ways of conceiving the central notions of semantics, truth and meaning, according to whether reality or the subject is the key factor involved in their explanation. We mentioned in passing idealist positions of the nature of objects, like those of Stewart Shapiro, Mark Heller or Diana Raffman, and we gave an overview of some representative pragmatist tenets. However, we adhered to a methodological canon instructing that, at the moment of adjudicating between rival theories, we should put metaphysics first. This led us to downplay the role of the context of utterance or the use of expressions. We resolutely espouse a worldly account of language rather than a subjective account of the world. Fuzziness in language could not be adequately explained by a non fuzzy, exact and precise world. We explain linguistic practices by the properties of objects, but not the other way around. Instead of making truth and meaning dependent on mind and use, I preferred to align myself with those who defend that the meaning of an expression is its denotation or reference, that facts are the meaning of sentences, and that it is universals that give being to particulars.

Concerning fuzziness itself, we examined three of its alleged characteristics. First, we saw that it was softly indeterminate, in the sense that a fuzzy object neither completely is *F* nor completely fail to be *F*. But this situation was distinguished from one of complete indeterminacy, from the total failure of the Principle of Excluded Middle. We previously had introduced at least three versions of that principle, and saw that, in its absolute form, the Principle of Exclusion of Intermediary Situations (PEIS), it had instances that were outright false, whereas in its weak form, it was completely true. Furthermore, both "$p \vee \sim p$", and "$p \vee \neg p$" are at least half true, but also at most half false when "p" is a borderline case. Second, we denied that fuzzy properties lacked any boundary, and explained that they had rather many boundaries, though of a mild, contradictory nature. And, in the third place, we explored what a borderline case is.

Two problems we were confronted with were the proper formulation of the major premise of the sorites paradox, and of the relation among the members of the soritical series. I contended that, in order to adequately capture both, one needed to use a weak negation, on pain of expressing the major premise in such a way that it was liable to be readily falsifiable, as it is the case with he conditional version of the major premise, the preservation principle. Moreover, the problem was how to characterize the soritical sequence so that the «distinctive indistinctness», the distinction without a difference between adjacent members was encoded in logical notation. The logical system here employed had the necessary resources to perform these two tasks better than classical logic. This was noted to be a first advantage of a non classical system that included among its logical vocabulary two different sorts of negation.

I also called the attention to the fact that the similarity among the contiguous members of the soritical sequence is what grounds the attribution of the fuzzy predicate to both or to none of them. Furthermore, the principle of fairness, enjoining to treat like cases alike, was invoked to lend additional support to the major premise, in the form of the *continuation principle* (that it is not the case that only one of a pair of indistinguishable members is *F*, while the other is not). And finally, the "intuitive" consideration that a grain does not make the difference between what is a heap and what is not was endorsed on the basis of the correlation between the underlying changes of a quantitative dimension, *G*, and the fuzzy property *F*, which supervenes on it (the *G-F* Correlation Principle).

If a number of reasons provide a foundation for the major premise of the sorites, the refusal of that premise will violate those same reasons serving as its foundation. We called the position denying the truth of the major premise, '*discontinuism*'.

The sorites was considered a fallacy, since the rule of inference used, Disjunctive Syllogism for weak negation, was invalid. However, our system also contains another version of DS for the strong negation, which is valid. So, the inferential power of classical logic is preserved.

The analysis of the special version of the slippery slope argument presented in Chapter 1 was particularly important in two respects. I defended it as a direct proof of its conclusion, that everything is *F*, and claimed that to renounce to its rule of inference amounted to embracing *maximalism*, the doctrine that only what is fully *F* is *F*. To avert this excess, I advocated a minimalism instead.

Finally, an historical appendix was added dealing with the conceptions of Heraclitus, Parmenides, Anaxagoras, Plato and Aristotle. The quick survey served the purpose of bringing to our attention the ontological side of the debate. It explored their ancient views on the relation among opposites, and on degrees of being and existence. It emphasized two different outlooks, that may well function as the ontological backgrounds of contemporary discussions of fuzziness: contradictorial gradualism vs. dichotomic discontinuism.

The next four chapters, from 2 to 5, presented the main ideas of the most prominent schools: agnosticism, supervaluationism, indeterminism, many-valued and fuzzy logics, intuitionism, nihilism, paraconsistent logics, and pragmatism. In most cases, more than one representative was considered. We examined the doctrines of at least twenty authors: Walton, Quine, Dummett, Williamson, Sorensen, Russell, Fine, Keefe, Tye, Rayme Engel, Goguen, Lakoff, Machina, Nicholas Smith, Wright, Unger, Horgan, Sylvan and Hyde, Vanackere, and Raffman.

Chapter 2 was devoted to agnosticism, the doctrine that a fuzzy sentence has one of the classical truth values, but that we cannot know which value actually it bears.

We first saw Quine's view that classical logic, and more specifically, its strong principle of bivalence, commanded that fuzzy words be made precise by an arbitrary stipulation. As a consequence, observational terms would have to be reconstrued as theoretical.

The chapter continued with a detailed discussion of the theory of Timothy Williamson, focusing on: (a) his claim that to deny the principle of bivalence is incoherent, (b) his proof that it is impossible to know where the sharp borderline lies, and (c) the supervenience of meaning on use. We recall that many critics objected to his attempt to ground a sharply bounded word on the use that people make of that word. In my personal assessment, I indicated that his proof of the inconsistency of non bivalent approaches was effective against radically *indeterminist* theories (those denying any form of the principle of excluded middle, and the weak version of the principle of bivalence), but failed to show any absurdity in the idea that a fuzzy sentence is neither completely true nor completely false, but has a non classical (designated and antidesignated) truth value. We also noticed that Williamson acknowledged that there is no cutoff point in nature, and that, therefore, fuzziness was dependent upon the subject's epistemological limitations. But, we commented that, if there was continuity in reality, we had better not postulate any sharp boundary. Furthermore, there were unsharpenable expressions, such as "somewhat tall".

Concerning the thought of Sorensen, we pointed out that he also maintained a subjectivistic attitude toward fuzziness for he said that the dilemmas of fuzziness sprang from our cognitive schemes rather than from the world. Moreover, for Sorensen, it is impossible to explain why the crisp border is located wherever it is. On the positive side, we appreciated his contention that contradictions are inescapable, but deplored that they were held only in thought, but not in reality.

Finally, I criticized both authors, Williamson and Sorensen, for (completely) falsifying the major premise of the sorites, and thus introducing a sharp cutoff, where there is none (discontinuism).

Chapter 3 dealt with supervaluationism and indeterminism. It exposed and criticized the thesis that a fuzzy sentence is neither true nor false.

It developed the supervaluationist idea that a borderline case of a predicate '*F*', initially indeterminate, could be made to fall either in the extension or in the antiextension of '*F*' by precisifying the meaning of the predicate; thus a fuzzy sentence could be made true or false. That this precisification is not incompatible with fuzziness was claimed to be achieved via the existence of several admissible precisifications, and vagueness just consisted in this multiplicity of precisifications. In order to keep the indeterminacy of borderline sentences and at the same time the principle of excluded middle, the truth-functionality of the connectives was relinquished. Truth was defined as truth in all admissible precisifications. Again, unfortunately, the paradox was solved by falsifying the major premise, since, in every precisification, there is a sharp cutoff. Supervaluationism was censured for its modifying the common understanding of the quantifiers, and its weakening of the Tarski schema.

Lastly, we saw Tye's different articulation of the indeterministic insight that sentences lack any truth value.

The main complaint against both views, the supervaluationist and indeterminist, was that the fact that, in a borderline case of *F*, it is plausible to go either way -as many indeterminist philosophers have acknowledged- does not entail that both alternatives cancel each other out. That is, if evidence is compatible with both, "p", and with "not p", then we are entitled to affirm both rather than being left in a situation in which we cannot affirm anything.

Chapter 4 presented the fuzzy and many-valued views.

It began with Rayme Engel's philosophical defence of the existence of gradual properties. He claimed that a definition of gradual property not mentioning degrees constitutes a distortion of the meaning of he corresponding predicate. Consequently, linguistic competence required the recognition of the different extents to which a property can be exemplified, since the issue of whether a fuzzy property is present or not is not a matter of all or nothing. Part of Engel's case rested on Leibniz' *principle of continuity*, that any change between opposites requires intermediate degrees. Without degrees, there was no continuous change.

Goguen argued that classical logic was inadequate to represent fuzzy properties , which should be assigned fuzzy sets as their referents, and that the set of truth values being the real numbers of the unit interval [0, 1] made it possible the continuity of the boundaries of fuzzy properties.

Lakoff sustained that one reason for assigning fuzzy sets as the denotations of fuzzy predicates was that there were best exemplars and peripheral members in a category, ordered in a hierarchy. This internal differentiation within categories induced degrees of membership in a set. Additionally, Lakoff contended that hedges, like 'very', could be appropriately modelled using degrees of truth.

However, both, Goguen and Lakoff, despite their admitting degrees of truth, were disapproved for their failing to uphold the principle of excluded middle and for espousing a maximalist conception of truth.

The chapter continued with Machina's exposition of why the graduality of borderline cases entails their contradictoriness, and why the appeal to degrees helped to explain the attractiveness of the major premise and the apparent validity of the argument. He claimed that classical logic should be extended to include intermediary degrees of truth, since fuzzy sentences do not obey strong bivalence, but are not indeterminate either. What was crucial in his analysis of the sorites was that at no step in the sequence of sentences a sudden change from 1 to 0 occurred.

The chapter ended with the new conception of Nicholas Smith, who proposed a definition of fuzziness in terms of *closeness*: that if two objects, x and y, are very similar in respect of the properties relevant to the application of a predicate *F*, then the sentences 'x is *F*' and 'y is *F*' are very similar in respect of truth. And he alleged that this definition required the existence of a continuum of truth values. There must be distinct truth values and yet very similar in their alethic status. As a result, a small difference in *F* relevant respects cannot produce a large difference in *F*. On the other hand, Smith rightly insisted on the cumulative effect of small differences. A change by means of a soritical series is possible.

Later on, Smith's replies to common objections against degrees of truth were described. The discussion finished with some considerations about how to determine, in the feasible cases, the exact truth value of fuzzy sentences, and with an admittance of agnosticism for the cases of unbounded properties.

Notwithstanding, Machina and Smith were criticized for allowing an argument with true premises but completely false conclusion to be valid.

Chapter 5 was a mixture of four remaining theories, which have had less popularity in the literature. We saw that Crispin Wright's intuitionistic conception of borderline cases as permissibility of faultless disagreement was upheld at the expense of there being a fact that sustained the conflicting opinions. Indeed, Wright renounced to the *truth maker principle*: that for every contingent true sentence, there is an existing fact in the world making it true. Additionally, Wright's conception of fuzziness agreed with the agnosticist one in not being at odds with the existence of exact boundaries. His solution of the sorites was achieved by invalidating double negation elimination, and by renouncing the objectivity of meaning. In borderline cases, matters are constitutively dependent upon us.

Peter Unger's and Terence Horgan's nihilist positions were then described. Both insisted that a fuzzy expression, '*F*', had contradictory features: it served to discriminate things that are *F* from those that are not, and it is applied to both or none of a pair of contiguous members in a soritical series that differ only minutely. There is no precise transition from *F* to not *F*. For Unger, the gradual nature of the world imposes no breaking point. And for Horgan, there is no minimal height for a person to be tall. I sympathized with this contradictorial conception of fuzziness, but disagreed on the consequences drawn from it. Both philosophers concluded that there were no fuzzy objects nor fuzzy properties in reality.

Unger went further in gathering that fuzzy terms are meaningless, and that, therefore, no sentence containing them was true. This skepticism resulted in language having less contact with reality. Horgan was worried about the problem of how a sentence predicating a fuzzy property of an object could be true if there were no fuzzy properties. He avoided the strict nihilist result concerning meaning and truth at the price of adopting a psychologistic semantics (terms are meaningful independently of a relation to something objective), and a conception of truth that did not include a direct relation between language and reality.

Subsequently, three paraconsistent systems were introduced: subvaluationism, Sylvan's and Hyde's relevantism, and Guido Vanackere's adaptive logic. All tried to accommodate the alleged contradictoriality of fuzziness by different techniques. Though the enterprise is really praiseworthy, I criticized them all for certain drawbacks pertaining to their particular approaches: the lost of the adjunction rule and the ordinary meaning of the universal quantifier, the failure of modus ponens to preserve truth, and an assimilation of the fuzzy word to an ambiguous expression, respectively.

Finally, Diana Raffman's contextualist proposal was presented and criticized. The attribution of a quality to an object was relative to the competent judge and the context. But, this eliminated the possibility of making unqualified predications. Besides, the relativization to the subject was unacceptable from a realist point of view.

After the display of the main proposals, Chapter 6 came to compare the two main contenders in the debate: discontinuism vs. contradictorial gradualism, and attempted to show the virtues of the latter. The phenomenon to be explained was the transition from one opposite to the other, by means of a soritical series: How is it possible that there is a transition if, for every pair of neighbouring members of the series, there is no borderline between them (*Continuation Principle*)? The strategy was to exploit the prima facie incompatibility between (1) the soritical series -as defined by the CP- and (2) the transition from F to not F: it was not possible to have both without contradiction. Fuzziness diffuses itself! While those theories that reject contradictions were forced to opt for (1) or (2), a paraconsistent system could tolerate such a contradictory combination and, thus, claimed to be superior to the alternatives in this respect.

Discontinuist positions denying the major premise of the sorites were objected for maintaining a notion of change that was reduced to a precipitous, abrupt, replacement of two stages, in clear violation of its apparent smoothness. Furthermore, the introduction of a sharp limit in the extension of predicates was equivalent to the elimination of proper borderline cases. An additional problem was pointed out, namely, the inability to satisfactorily explain the transition question: it occurred arbitrarily, without a foundation in the subvening quantitative dimension G. The transition from heap to non heap would take place prompted by a small change in one grain! That was incredible.

At the end, the contradictorial and gradualist solution was explained in detail. It had the virtue of reconciliating both the soritical series and the transition. Most importantly, it kept the truth of the major premise, doing justice to the continuity (that there is no borderline between undistinguishable things), definitory of fuzziness. The existence of degrees of truth was suggested to follow from that of degrees of possessing a property, by means of the generalized redundancy truth principle. And an account was offered of how the transition is effected: as one opposite begins to gradually fade out, the other starts manifesting itself. Throughout the transition, there is a blend or mixture of opposites, but in different proportions. This overlap was only possible because each opposite is just partially present, so that it can coexist with its contrary, which is also present only to some non maximal extent. Thus, the graduality of properties resulted in contradictory situations. The transition happens through intermediate stages, as Leibniz said.

All things considered, it seems that the account here advocated is less vulnerable to the difficulties that we have encountered in other theories. I deem one of its main merits is that it manages to make sense of the graduality and contradictoriality, which we believe are the marks of fuzziness. Thus, real fuzziness has been vindicated! Moreover, the sorites paradox has been solved without a sharp boundary. Fuzziness does support its major premise.

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